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The manuals on electronic fundamentals form a progressive series of educational texts which present the theory and application of electronics for the military services. The series starts with the basic electrical fundamentals and extends to the most recent technical concepts, as applied to telegraphy, telephony, radio, loran, facsimile, radio direction finding, radar, meteorological radio, television, and other military equipments. The entire series, all of which will be published within the succeeding 12 months, will be given the same basic distribution. The numbers and titles of these manuals are as follows:

Introduction to Electronics	TM 11–660
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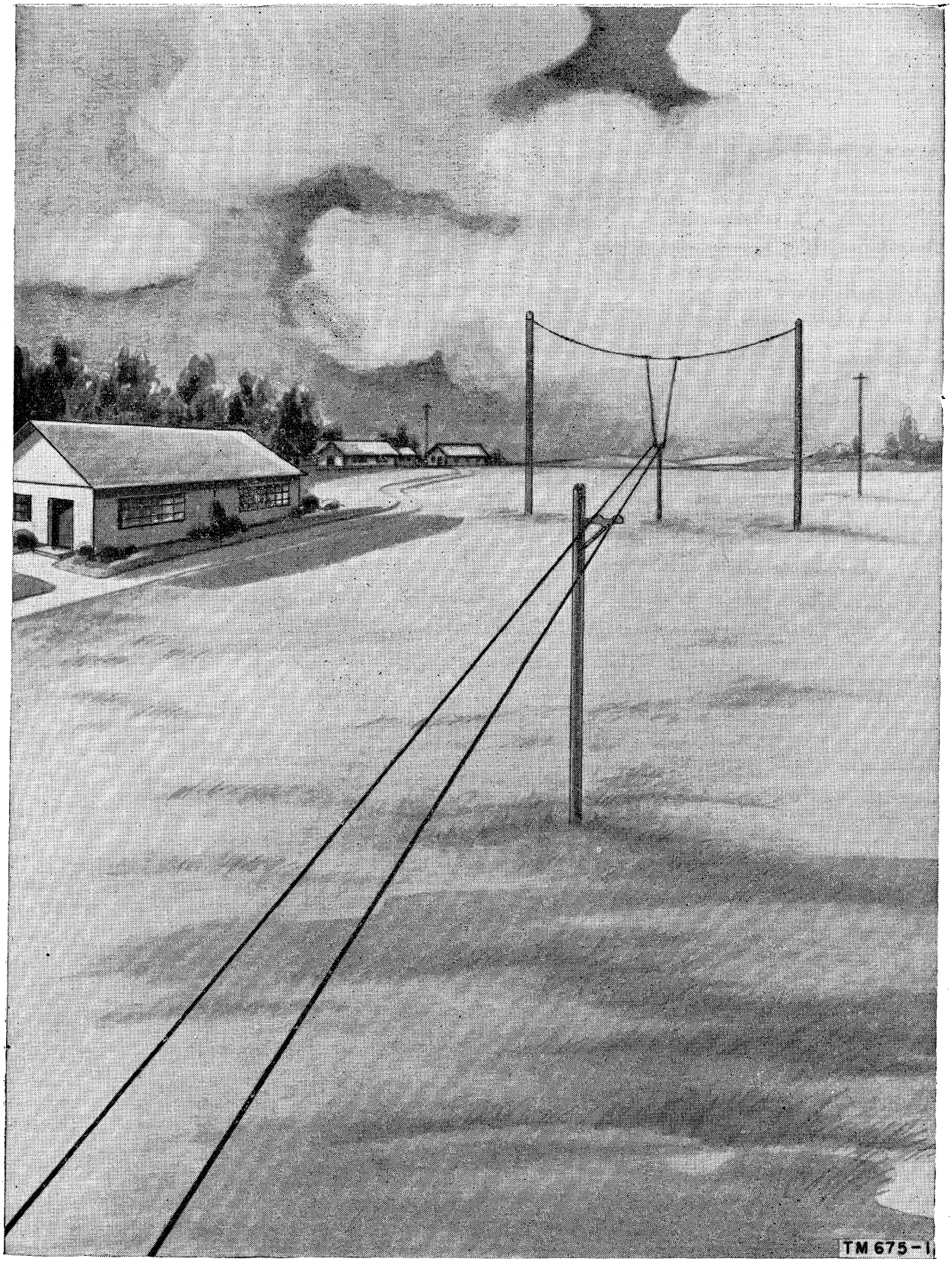

Figure 1. Electricity serves as a connecting link.

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CHAPTER 1

FUNDAMENTAL LINE THEORY

Section I. INTRODUCTION

1. Electricity as a Link

With the throwing of a switch or the pushing of a button, lights go on, factory machinery hums, or a bridge opens or closes. Because of this ease of application to his needs, man is likely to consider electricity as an original or a primary source of power. In every instance, however, work must first be done to convert other kinds of energy—the heat energy of coal and other fuels, the kinetic energy of rivers, waterfalls, and tides—to electric energy. One of the principal advantages of electric energy is that it can be transmitted efficiently over networks of conductors, called *transmission lines.* In other words, electricity serves as a *connecting link* between a primary source of energy, such as a coal mine, and distant machinery or other devices that will do the actual work (fig. 2).

2. Transmission Lines and Waveguides

a. TRANSMISSION LINES. A transmission line may be defined as a line used for electric power transmission. At first glance, it might seem that this definition covers only lines used to distribute electric power from huge power plants to cities and communities many miles distant. The definition is, however, equally applicable to telephone, telegraph, and radio lines. Large electric power plants may deliver *thousands of kilowatts* of power to factories for operating huge machines, to communities for street lighting, and to homes for lighting and for operating washing machines and other labor-saving devices. A telegraph or telephone line may transmit *less than 1 milliwatt* of power, the information being carried by *variations* in frequency, phase, or amplitude. Radio lines (fig. 1) may carry many *kilowatts* of power from a transmitter to an antenna, or *microwatts* of power from an antenna to a receiver. In each instance, electric power is transmitted, but the *amount* of power and its ultimate use may differ considerably.

b. WAVEGUIDES. Strictly speaking, all transmission lines (par. 3) are waveguides, because the greater portion (about 90 percent) of the transmitted energy is in the electromagnetic waves that the line conductors *guide* through the space *between* them. In general, less than 10 percent of the transmitted energy is actually *in* the conductors of which a well-designed line is comprised. In practice, however, the term *waveguide* usually means a device such as a hollow metallic conductor or solid rod of nonconducting material within which electromagnetic waves are guided from one point to another (fig. 6). Transmission lines using spaced conductors are not commonly known as waveguides and may be explained more or less adequately in terms of conductor currents and voltages. In this text, only *radio-frequency* transmission lines for communications purposes will be considered.

3. Types of Transmission Lines

Transmission lines may be broadly classified according to their use as power or communications lines. Within each of these classifications, lines may be further described in terms of a particular characteristic or use; for example, high tension lines in power circuits, long lines in telephone circuits, and tuned lines in radio circuits. In accordance with certain structural details and regardless of its particular application, a given line may be accurately de-

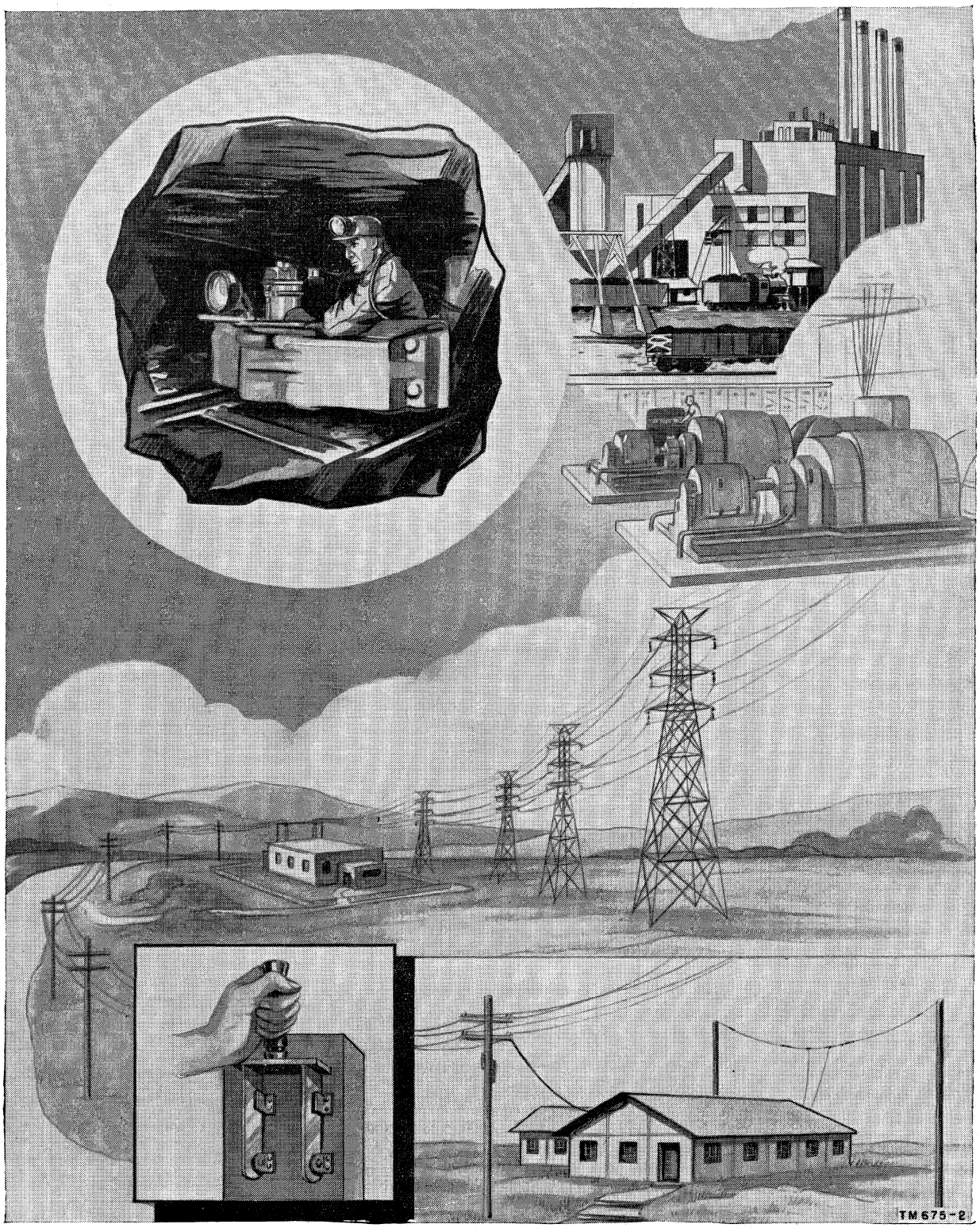


Figure 2. Lines carry power from source to user.

scribed as a single-wire, multiwire, or coaxial line.

a. SINGLE-WIRE OR GROUNDED LINE. Historically, the invention of the telegraph developed the first need for extensive electrical conducting systems. Because it had already been discovered that the earth could be used as a return conductor of electricity, and because of the considerably greater economy, the first telegraph lines were single-wire, grounded lines (fig. 3). Although single-wire lines were adequate for the early telegraph systems, harsh, unexplainable noises made conversations difficult or impossible when these lines were used to interconnect telephone instruments. When a number of single-wire telephone lines were run close together, the conversation on one line could be heard on all the other lines (cross-talk). With the expansion of power circuits and the building of electric railways, both of which were usually close to the telephone lines, single-wire telephone lines became practically useless because of noise currents entering them from the power and railway circuits. Because of these troubles, single-wire lines are not used as telephone lines today, except in emergencies or where the added expense of more efficient lines is not justified. However, these lines are still used in certain telegraph and radio circuits.

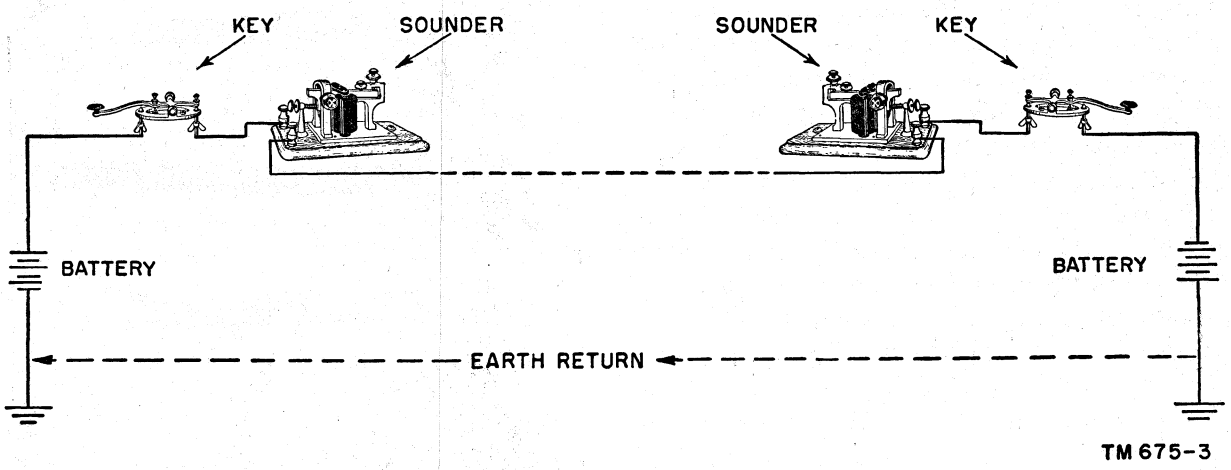


Figure 3. Earth used as return conductor.

b. TWO-WIRE LINES (fig. 4). Years of cut and dry experimentation, together with the development of excellent mathematical analyses of line operation, proved that a metallic return is the only cure for noise and cross-talk troubles. The need for a metallic return led to the construction of two-wire lines consisting of two metallic conductors of the same material and spaced the same average distance throughout their length. When bare wires are used in such a line, the spacing must be great enough to prevent contact between the wires. As will be explained throughout the text, the conductor size, type of dielectric material, and spacing all affect line characteristics and must be chosen carefully (fig. 5). The principal advantages of two-wire lines are their greater freedom from interference from external circuits and considerably higher efficiency due to lower losses. Because of these and other advantages to be discussed later, two-wire lines are used extensively in radio as well as in telegraph and telephone circuits. It might be well to mention here that a detailed study of two-wire lines forms a basis for the study of coaxial lines, waveguides, and antennas.

c. THREE- AND FOUR-WIRE LINES. As their names imply, these lines are similar in construction to the two-wire line, but have additional conductors. Multiwire lines are used in polyphase power circuits and may be used in telephone and radio circuits to provide improved

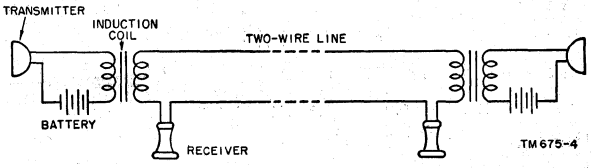


Figure 4. Metallic return of two-wire line is more efficient.

shielding and other advantages at a cost lower than that of coaxial lines.

d. COAXIAL LINES. A coaxial line, usually called a coaxial cable, consists of a cylindrical conducting rod or wire within and concentric with an outer, hollow cylindrical conductor (fig. 5). Owing to the perfect shielding of the inner conductor by the outer conductor, the transmitted energy is confined to the space between the inner conductor and the inner wall of the outer conductor; that is, none of the transmitted energy appears on the outer surface of the cable. Under certain conditions of operation, undesirable currents, called surface currents, may flow on the outer surface of the cable, and steps must be taken to prevent or reduce these currents to a minimum. Coaxial cables are particularly useful as r-f (radio-frequency) transmission lines, partly because the shielding prevents radiation from the line and partly because of the broad band of frequencies that can be transmitted over these lines with little or no frequency distortion. Ordinary telephone lines transmit a relatively narrow frequency band extending from about 150 cycles to 2,500 cycles; special equalized telephone lines handle a somewhat broader frequency band of 60 cycles to approximately 10,000 cycles; but a properly designed coaxial cable transmits with negligible frequency distortion a frequency band approximately 3 mc (megacycles) in width. Thus, special coaxial cables are used to interconnect television stations in networks similar to those used for broadcasting speech and music.

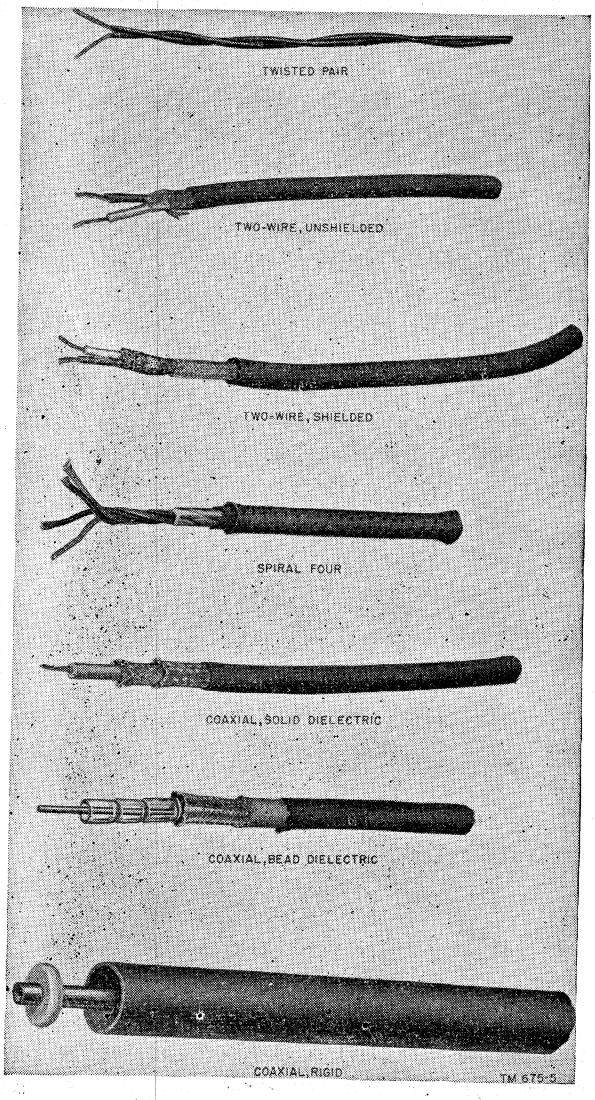


Figure 5. Types of transmission lines.

4. Line Applications

Although overhead power and communications lines spread across the nation like a gigantic spiderweb (fig. 7), the average person is scarcely aware of their existence, accepting them—along with trees, billboards, and hot-dog stands—as part of the scenery. But these lines are far more than ugly blots on the landscape; they make possible the telephone, the radio, the television, the washing machine, and thousands of other conveniences in the home. To power and communications engineers, these lines represent years of research and labor that led not only to lines that carry electric energy from point to point, but also to lines that *act* as circuit elements such as capacitors, inductances, and resistances. Thus—apparently in violation of certain fundamental electrical laws—transmission lines, constructed of very nearly perfect *conducting* materials, act as *insulators,* and hollow pipes (fig. 6) guide electromagnetic waves!

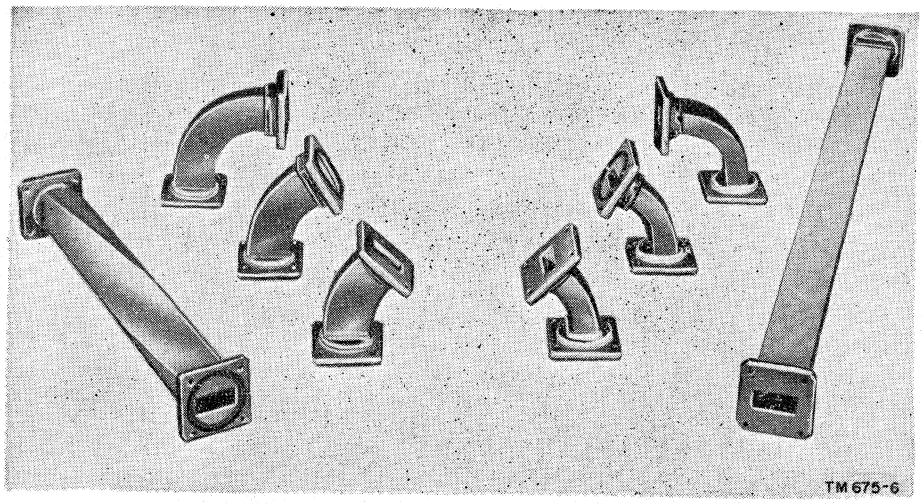


Figure 6. Typical waveguide sections.

5. Line Theory

Every transmission line operates in accordance with the fundamental laws of electricity and magnetism (TM's 11–661 and 11–681). However, because they are intended for the beginning student, these manuals necessarily are limited in scope and cover only the more elementary applications and aspects of the fundamental laws. Some of the more advanced concepts, particularly field concepts, are presented in the following sections of this chapter. Before beginning the study of transmission line theory, students should review the elementary information in TM 11–661 and TM 11–681.

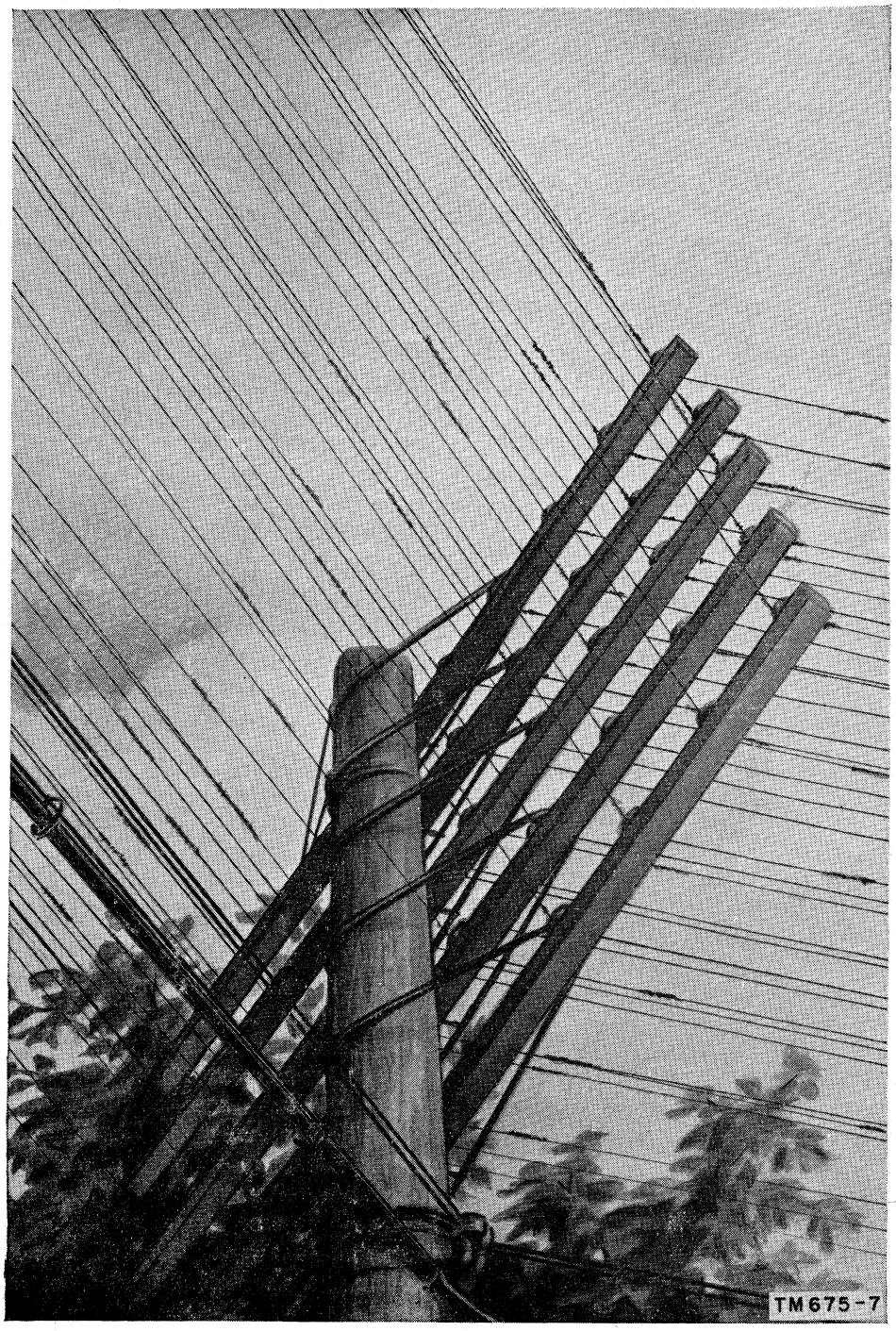


Figure 7. Transmission lines spread across the nation.

Section II. TRANSMISSION LINE PROPERTIES AND ELECTRICAL FUNDAMENTALS

6. Circuit Properties

Transmission lines possess the three inherent characteristics or properties of all electrical circuits: resistance, R; inductance, L; and capacitance, C. It is a natural law that all three electrical properties exist together and, therefore, it is impossible to design any circuit element to possess only one or two of these characteristics.

a. LUMPED AND DISTRIBUTED PROPERTIES. A circuit element can be designed to act as a relatively large quantity, or lump, of any one electrical property. This is done by reducing the other two properties to minimum values, so that their effects are negligible *at the operating frequency of the circuit.* In A, figure 8, the inductance is lumped as a relatively large quantity in coil L, and the other characteristics, represented by R and C, are distributed throughout the coil and reduced to minimum values. Capacitance C (B, fig. 8) is a lumped capacitance which necessarily possesses small distributed quantities of the other two electrical properties; resistance is present as shunt resistance R_1 and as series resistance R_2; the inductance L is due chiefly to the lead wires. Resistor R (C, fig. 8) also has small distributed quantities of inductance L and capacitance C, as well as its lumped quantity of resistance. At frequencies below which their distributed properties become important, coil L, capacitor C, and resistor R act as an inductance, a capacitance, and a resistance, respectively. At higher frequencies, the coil may act as a capacitance or resistance, the capacitor as an inductance or resistance, and the resistor as an inductance or capacitance.

b. TRANSMISSION LINES AS CIRCUIT COMPONENTS. Capacitors, inductance coils, and resistors are thought of as separate components which are inserted in an electrical circuit to place or localize definite quantities or lumps of capacitance, inductance, or resistance, according to the circuit requirements at the point of insertion. Similarly, transmission lines may be regarded in certain applications as *components* which are inserted in an electrical circuit for the purpose of producing the desired operation of the circuit as a whole. Transmission lines differ from other circuit components chiefly in the distribution of the electrical properties, L, C, and R; that is, L, C, and R are not present as localized or lumped quantities, but each is distributed evenly throughout the length of the line.

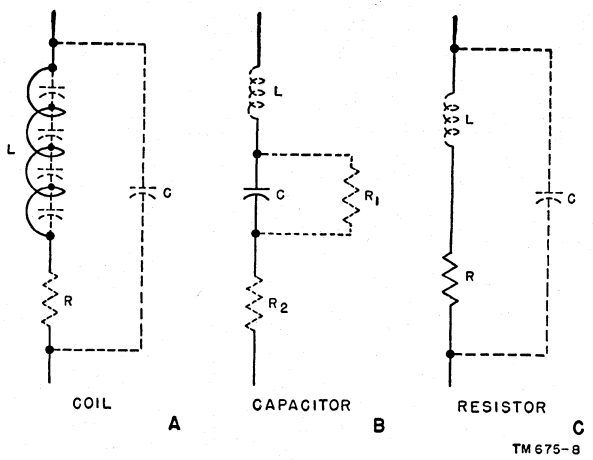


Figure 8. Every circuit has R, L, and C.

c. SCHEMATIC REPRESENTATION. The simple schematic in A, figure 9 represents a generator connected to a series circuit comprised of a coil L, a capacitor C, and a resistance R. The inductance, the capacitance, and the resistance are lumped quantities. In B, figure 9 a two-wire transmission line, comprised of two parallel wires, is connected to the generator terminals. In order to include the inductance and capacitance of such a line, the schematic, B, is redrawn in C, figure 9. This illustration shows that the inductance, represented by the dotted turns about each wire, and the capacitance, represented by the series of dotted capacitors between the wires, are *distributed* throughout the entire length of the line. A transmission line not only contains distributed inductance and capacitance as illustrated in C, figure 9 but also includes series resistance R and shunt conductance G. The series resistance is due to the inherent opposition of the wire to a flow of current, and the shunt conductance is due to the imperfect insulating quality of the medium between the wires. To include

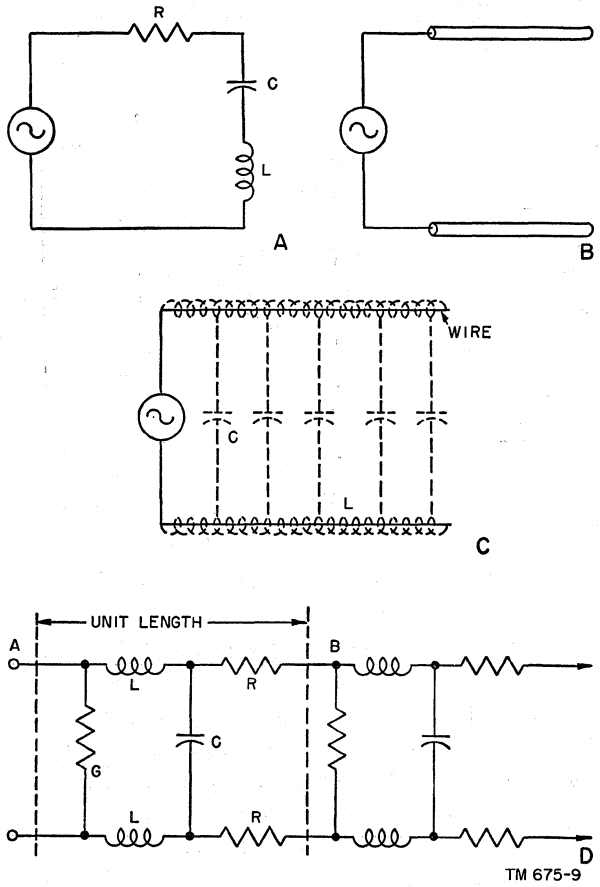


Figure 9. R, L, C, and G are distributed along a transmission line.

the series resistance and the shunt conductance in the schematic, the diagram of C, figure 9 may be redrawn as shown in D.

d. UNIT LENGTH. In dealing with transmission line theory, it is convenient to consider each line in terms of unit lengths; that is, the line is divided into small lengths each of which is known as a unit length. In D, figure 9, the portion of the line between *A* and *B* is one complete unit length; *G* represents its shunt conductance, *C* is the distributed capacitance, *L* is the distributed inductance, and *R* is the series resistance. Each unit length is identical to every other unit length throughout a line.

e. GENERAL CONSIDERATIONS. Resistance and shunt conductance cause a waste of power within a transmission line. In general, therefore, transmission lines are designed so that their resistance and shunt conductance are reduced to the minimum possible values. In well-designed r-f lines the resistance and shunt conductance are so low that, for most purposes, they may be neglected. Of course, there are some r-f line applications in which the resistance, minute as it may be, forms a basis for the action of the line.

(1) It might be well to note at this time that all transmission lines do not have the even distribution shown in D, figure 9. The submarine cable and certain telephone cables, such as the spiral four (fig. 5) are excellent examples of this; they generally have large capacitance and relatively little inductance. In order to overcome this, coils (lumped inductances) may be inserted at intervals in the cable, adding or loading the line with the necessary inductance but producing an uneven distribution.

(2) Whether composed of lumped quantities of *L, C,* and *R,* as in A, figure 9 or composed of distributed quantities of *L, C,* and *R,* as in D, figure 9, an electrical circuit always operates in accordance with certain basic electrical phenomena. In dealing with low-frequency circuits made up of lumped values of *L, C,* and *R,* it is possible to explain their action in terms of voltage and current, without reference to the more basic concepts of electric and magnetic fields. In order to deal intelligently with circuits of the type illustrated in D, figure 9, however, the student should possess a more complete knowledge of the basic electrical theories. For this reason, paragraphs 7 through 22 and section III of this chapter are devoted to a discussion of electrical concepts that lead to a better understanding of transmission lines and coaxial cables.

7. Kinds of Electricity, Electric Charge

Electricity consists of two elementary particles, arbitrarily called positive and negative. The elementary positively charged particle is called a proton; the elementary negatively

charged particle is called an electron (fig. 10). The proton and electron are *elemental electric* charges and, although the mass of the proton is about 1,800 times greater than that of the electron, the proton and electron are *equal quantities* of *opposite kinds* of electricity.

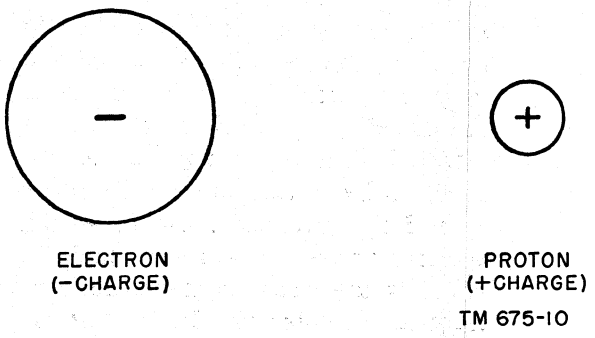


Figure 10. The electron and the proton are equal quantities of opposite kinds of electricity.

a. ELECTRIC FORCE. The behavior of every electrical circuit, from the simplest to the most complex, is due to the electric force that exists in the *space* about every elemental charge. This electric force *produces action on other charges at a distance;* like charges repel one another, unlike charges attract one another. As indicated in A, figure 11, the force between two positive charges pushes them apart; the force between two negative charges pushes them apart, as in B, but the force between two unlike charges pulls them together, as in C.

b. CHARGED BODIES.

(1) *Neutral condition.* The atoms of all normal material bodies are thought to contain equal numbers of individual negative charges (electrons) and positive charges (protons). Because the electric force of an electron is equal and opposite to that of a proton, the equal numbers of electrons and protons in a normal body produce no action on other electric charges at a distance. In other words, the electric force due to one kind of charge is neutralized by the electric force due to the opposite kind of charge, and the body is said to be neutral or uncharged.

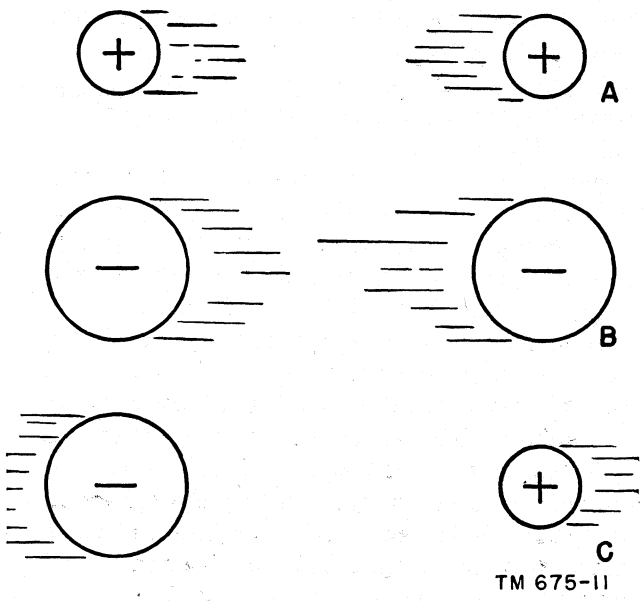


Figure 11. Like charges repel one another, A and B; unlike charges attract, C.

(2) *Charged condition.* A body is said to to be electrically charged whenever the number of its electrons is not equal to the number of its protons; the excess of one or the other constitutes an electric charge. Hence, a body is negatively charged whenever electrons have been *added* to it, so that the number of its electrons exceeds its normal number of protons. The body is positively charged whenever its normal number of electrons has been reduced, so that the number of its protons exceeds that of its electrons.

8. Electric Field, Lines of Force

It has been stated that the forces acting between electric charges are called electric forces. The *space* surrounding an electric charge is called an *electric field;* that is, an electric field is the *space* in which the electric force about a charge is present to act on other charges that may be within the space. However, for simplicity, the space is given the property of the force within it; that is, an electric field is thought of as the *force* existing at every point in the space about an electric charge.

a. DIRECTION OF FIELD. The direction of an electric field is arbitrarily defined as the direc-

tion in which the field tends to move a positive charge. Lines drawn to represent these directions are called electric lines of force. In A, figure 12, the lines of force represent the directions of the field about a positively charged sphere; the lines of force about a negatively charged sphere are shown in B, figure 12. Although drawn only in the plane of the paper, lines of force actually extend in *every* direction. If a positively charged body, *C*, is placed in the field about the positively charged sphere it will be urged away from the sphere; the same field will attract a negatively charged body, *D*, to the sphere. The short arrow on each of the inserted charged bodies indicates the direction in which each is urged. Because the sphere in B, figure 12, is negatively charged, it attracts the positively charged body, *E*, toward it and repels the negatively charged body, *G*.

b. FUNDAMENTAL CHARACTERISTICS. *An electric charge, whether positive or negative, cannot exist without its electric field; the charge and its field cannot be separated. The electric field of every charge extends to infinity, exerts forces on all electric charges within it, and possesses inertia. Because a charge and its field are inseparable, whenever a charge is in motion, its electric field also must be in motion.*

c. RESULTANT FIELDS.

(1) *Zero field.* If equal opposite charges are placed together (fig. 13), their individual fields are coexistent and equal; neither field is made nonexistent by the other. However, the electric field of the positive charge (solid arrows) is in a direction opposite to that of the negative charge (broken arrows). Consequently, the electric field of the positive charge will urge an inserted positive charge *A* in the direction of force f_1 (solid arrow), and the oppositely directed field of the negative charge will *simultaneously* urge the inserted positive charge in the direction f_2 (broken arrow). An inserted negative charge *B* is also acted on by equal and opposite forces f_1 and f_2. Because these oppositely directed forces are exactly equal, the *effect* is the same as if neither field existed. In other words, the coexistent, equal, and opposite fields produce *no action* on inserted charges and the *resultant* electric field is said to be zero. (In practice, the effects of the fields of inserted charges cannot be neglected. However, it is assumed here that the inserted charges are so small that their fields are negligible.)

(2) *Charges in a line of infinite length.* If a number of positive charges are placed in a line of *infinite* length A, (fig. 14), the electric fields of the individual charges are coexistent and combine to produce a resultant field *the direction of which is everywhere perpendicular to the line of charges.*

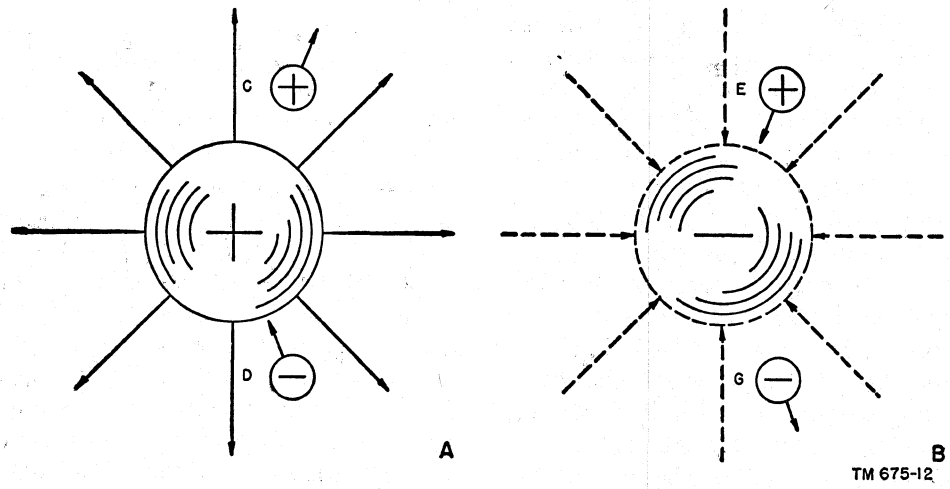


Figure 12. Electric lines of force indicate direction in which a plus (+) charge is urged by the field.

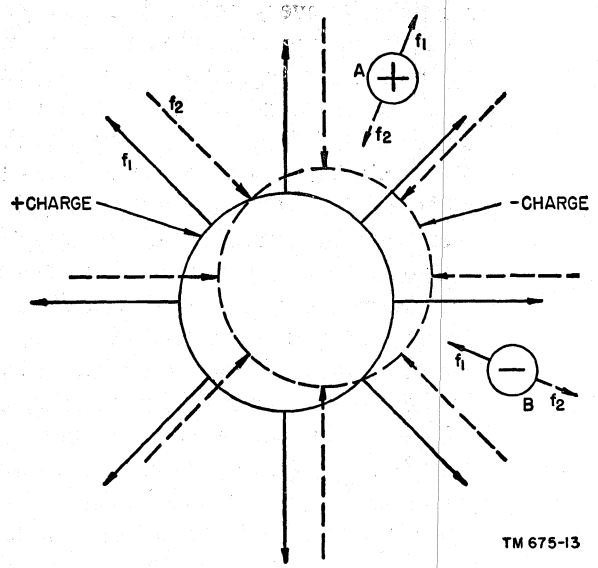


Figure 13. Fields of individual charges always exist, but may combine to produce a zero resultant field.

A similar line of negative charges is surrounded by a similar resultant field, except that the direction of the force is opposite B, (fig. 14). An uncharged conductor may be considered in terms of equal numbers of *equal* positive and negative charges alternately arranged in a line. A, figure 15, shows, crudely and in only one plane, that the fields of the positive, f_1, and negative, f_2, charges are superimposed (occupy the same space); therefore, the resultant field in the space is as though neither of the oppositely directed fields existed. For simplicity, an uncharged body is pictured without any lines of force B, (fig. 15); it is rarely necessary to show the elemental fields that combine to produce a resultant field.

(3) *Representation of fields.* In this manual, unless otherwise stated, lines of force are always drawn to represent the *resultant field.* Thus, in A, figure 14, the lines of force represent the *resultant* field due to the excess number of positive charges in the conductor; the lines of force in B represent the field *resulting* from the excess number of negative charges. Compare A, figure 14 and A, figure 15 and note

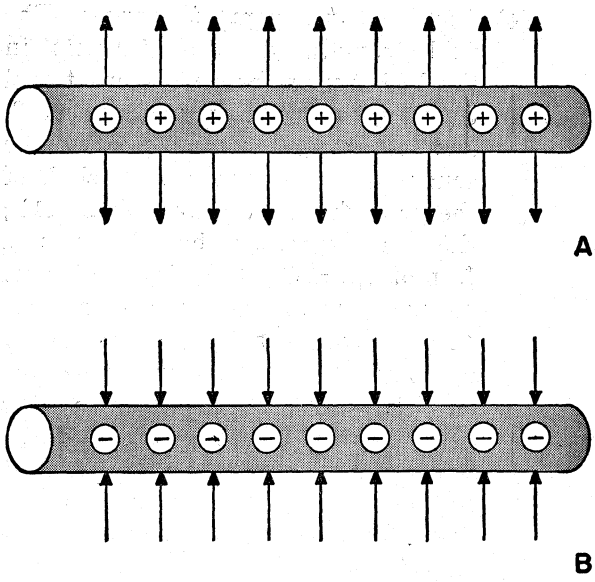


Figure 14. A long line of + charges, A, or a long line of — charges, B, produces an electric field which is everywhere perpendicular to the row.

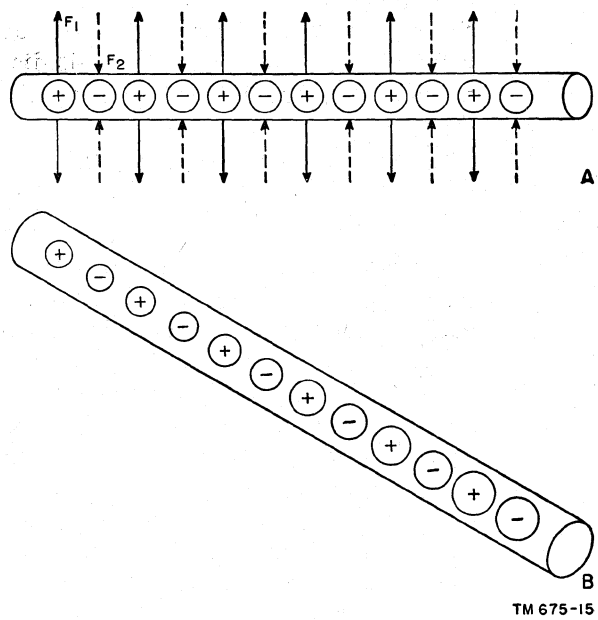


Figure 15. Opposite equal fields occupy the same space, A, and the resultant field is zero, B.

the simplicity obtained in the former by considering only the excess charges in the conductor. It should be noted here that, although spoken of as though they were real, lines of force are imaginary and represent only a *condition of the space* through which they are drawn.

(4) *Field about separated charges.* The field strength, symbol S, at a point in the field about a charge is proportional to the quantity of charge, symbol Q, and inversely proportional to the square of the distance, symbol d, of the point from the charge (par. 11); this is expressed by the simple formula: $S = Q/d^2$. In figure 16 equal charges A and B are *not* superimposed as in figure 13, but are separated by a distance A–B. The arrow on line A–X represents the direction that the electric field of the positive charge will urge a positive charge placed at any point along A–X. The arrow on line Y–B shows the direction that the electric field of the negative charge will urge a positive charge placed at any point along Y–B. In A, figure 16, the quantity QA of positive charge at A equals the quantity QB of negative charge at B, A–X, crosses Y–B at point C, and distance A–C equals distance C–B. From the field strength formula,

$$S1 = Q_A/d^2 = Q_B/d^2 = S2$$

In words: At point C the field strength $S1$, due to the positive charge at A, equals the field strength $S2$, due to the negative charge at B. Hence, a positive charge placed at C is simultaneously urged by a force f_1 from C toward X and by an equal force f_2 from C toward B. Obviously, the charge cannot move in two directions at the same time, but forces f_1 and f_2 acting together produce at C a resultant force f_R in the direction C–E. Consider now the condition of the field at point D in B, figure 16. Distance A–D is greater than distance D–B and, therefore, force f_1 is *less* than force f_2. Under this condition the resultant force f_R at point D is downward and slightly to the right. If a free plus charge is similarly placed at numerous points in the combined fields of charges A and B, it will be found that the superimposition of the individual fields produces a *different* resultant condition at every point. Thus, a free positive charge, placed at point C in figure 17, comes under the influence of a *different* resultant force as it leaves C and follows the curved path C–B. Similarly, a plus charge placed at D will follow the curved path D–B. Note that the curved resultant line of force A–C–B is perpendicular *at its ends* to the surfaces of A and B. In figures 17 and 18 a few of the elemental lines of force are indicated by broken lines. The solid lines in figure 17 represent the resultant lines of force in the electric field about two equal and *unlike* charges; the solid lines in figure 18 represent the resultant lines of force about two equal and *like* charges. As used in these figures, each line of force merely indicates that a force acts in the direction of the line on a positive charge placed at any point on the line, and that the force acts in the opposite direction on a negative charge.

9. Electrical Units

Up to this point only elemental charges have been mentioned as a unit quantity of electricity, but in order to deal intelligently with practical work, it is necessary to establish other electrical units. The student is undoubtedly familiar with the practical units, such as the volt, ampere, ohm, farad, and so on. These practical units were developed from more basic units. When first used in this manual, units other than practical units will be defined.

a. UNIT OF FORCE. The *dyne* is a commonly used unit of force and is defined as the force which, acting on a mass of 1 gram (.035 ounce) for 1 second, increases its velocity by 1 centimeter per second.

b. ELECTROSTATIC UNIT QUANTITY OF ELECTRICITY. This is defined as that quantity of electricity which, when concentrated at a point in a vacuum, repels an equal quantity of like charge at a distance of 1 centimeter (.3937 inch) with a force of 1 dyne, or attracts an unlike charge with the same force.

c. THE COULOMB. The electrostatic quantity

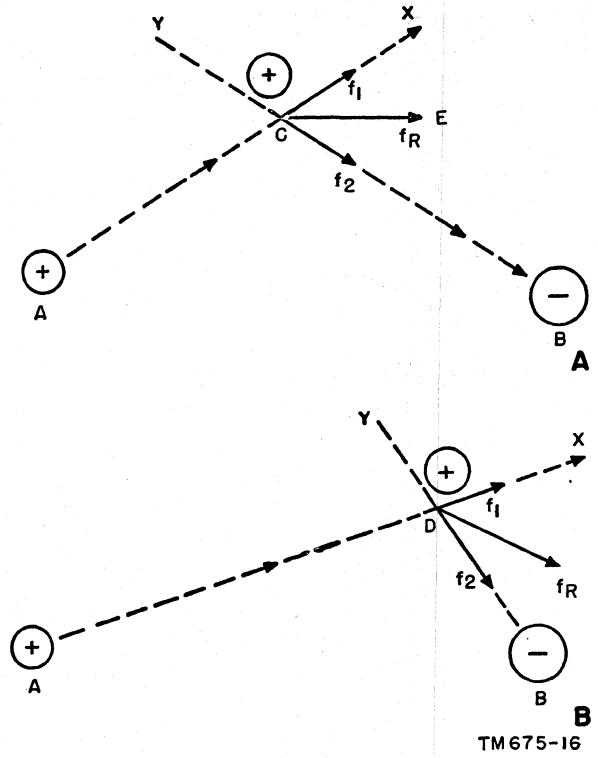


Figure 16. Direction of resultant field is a function of every individual field at each point.

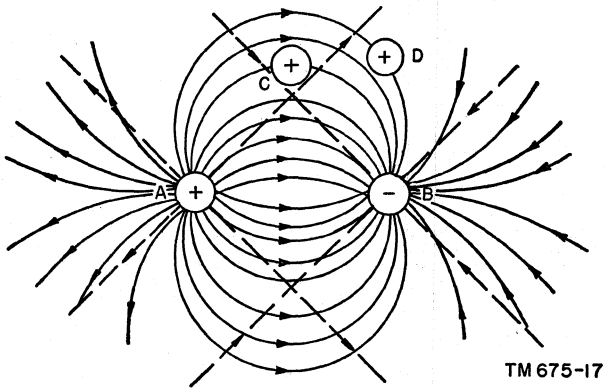


Figure 17. Field about equal unlike point charges.

of electricity is called a statcoulomb. The number of elemental charges in a statcoulomb has been calculated to be greater than 2,000,000,000! Even this huge number of elemental charges provides such a very small unit that it is not used in practical electricity; the equivalent practical unit is the coulomb, and 1 coulomb is equal to 3 x 10^9 statcoulombs or *3,000,000,000* statcoulombs. In terms of elemental charges the coulomb is equivalent to 6.28 x 10^{18} electrons (or protons). Although not used as frequently

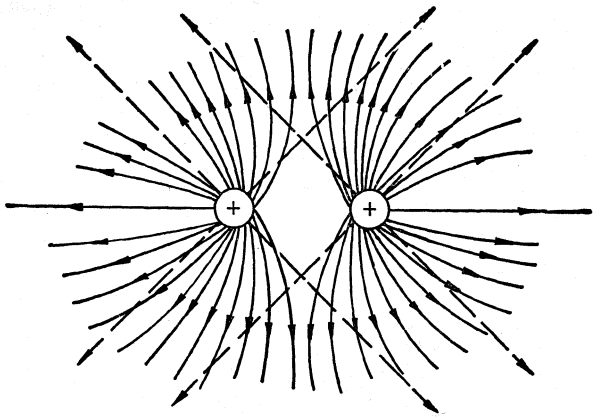


Figure 18. Field about equal like point charges.

as the volt, ampere, and ohm, the coulomb is a practical unit and is a measure of *quantity*—like gallons and bushels. The relationship of the coulomb to other units may be clearer when it is realized that if 1 coulomb of electricity is passing a point in a circuit in 1 second the current is 1 ampere. Thus, when the current in amperes is known, the number of electrons passing any point in a circuit can be calculated.

10. Force Between Charges

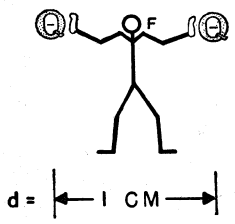
Experiment has shown that the force between two charges (in a vacuum) varies inversely as the square of the distance between them and directly as the product of the two quantities of charge. Thus, regardless of the kind of charge, the combined effect of quantity and distance is expressed by a simple equation:

$$F = Q1Q2/d^2$$

where:
F = Force between the two charges
$Q1$ = Quantity of one charge
$Q2$ = Quantity of the other charge
d = Distance between $Q1$ and $Q2$

For most practical purposes, the force existing between charges in air is considered equal to the force existing between charges in a vacuum. If $Q1$ and $Q2$ are like charges, the force between them is one of mutual repulsion; if $Q1$ and $Q2$ are unlike charges, the force between them is one of mutual attraction. Use of the force formula is illustrated in figure 19, where the figure F represents the force between charges.

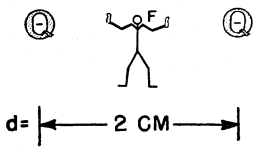
(A)



$-Q_1 = -Q_2 = 1$ UNIT CHARGE
$d = 1$ CENTIMETER

$$F = \frac{Q_1 \times Q_2}{d^2} = \frac{1 \times 1}{(1)^2} = 1 \text{ DYNE}$$

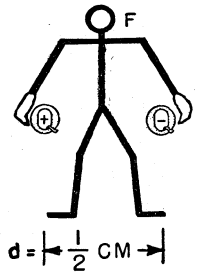
(B)



$-Q_1 = -Q_2 = 1$ UNIT CHARGE
$d = 2$ CENTIMETERS

$$F = \frac{Q_1 \times Q_2}{d^2} = \frac{1 \times 1}{(2)^2} = \frac{1}{4} \text{ DYNE}$$

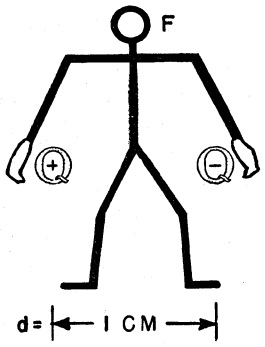
(C)



$+Q_1 = -Q_2 = 1$ UNIT CHARGE
$d = 1/2$ CENTIMETER

$$F = \frac{Q_1 \times Q_2}{d^2} = \frac{1 \times 1}{(1/2)^2} = \frac{1}{1/4} = 4 \text{ DYNES}$$

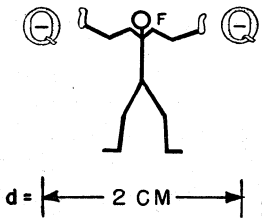
(D)



$+Q_1 = 2$ UNIT CHARGES
$-Q_2 = 3$ UNIT CHARGES
$d = 1$ CENTIMETER

$$F = \frac{Q_1 \times Q_2}{d^2} = \frac{2 \times 3}{(1)^2} = 6 \text{ DYNES}$$

(E)



$-Q_1 = 2$ UNIT CHARGES
$-Q_2 = 3$ UNIT CHARGES
$d = 2$ CENTIMETERS

$$F = \frac{Q_1 \times Q_2}{d^2} = \frac{2 \times 3}{(2)^2} = \frac{6}{4} = 1.5 \text{ DYNES}$$

TM675-19

Figure 19. Force between charges is readily calculated.

11. Electric Field Strength

a. DEFINITION. The electric field strength (intensity) at any point in the field about a charge is defined as *the force which would act on a unit positive charge (placed as a test charge) at that point in the field.* This definition comes from the basic force equation: $F = Q1Q2/d^2$. With field strength defined as the force acting on a *unit* positive charge placed at a point in the field, the field strength at various points in the space about one charge is determined by maintaining a second charge $Q2$ at unit value and moving it as a test charge at various distances from the first charge $Q1$. Thus, in calculating field strength, $Q2$ is always numerically equal to 1 and, consequently, it can be dropped from the equation. The equation for field strength, symbol S, at various points in the field about any charge then becomes $S = Q1/d^2$. The illustrations in figure 20 show the application of the field strength formula in determining the field strength at points in the field about different quantities of charge. Note that the figure S is shown as acting only on the inserted test charge Q_A. This is because *field strength* is the *force at a point,* not the force between charges.

b. TOTAL FORCE AT A POINT. If the field strength at a point is known, *the total force at that point,* when the inserted charge is greater or less than unity, can be found by multiplying the field strength by the quantity of inserted charge:

$$F = SQ2$$

where: F = Force
S = Field strength at point of insertion
$Q2$ = Quantity of inserted charge

The illustrations in figure 21 show how the total force is calculated when only the field strength and the quantity of the inserted charge $Q2$ are known.

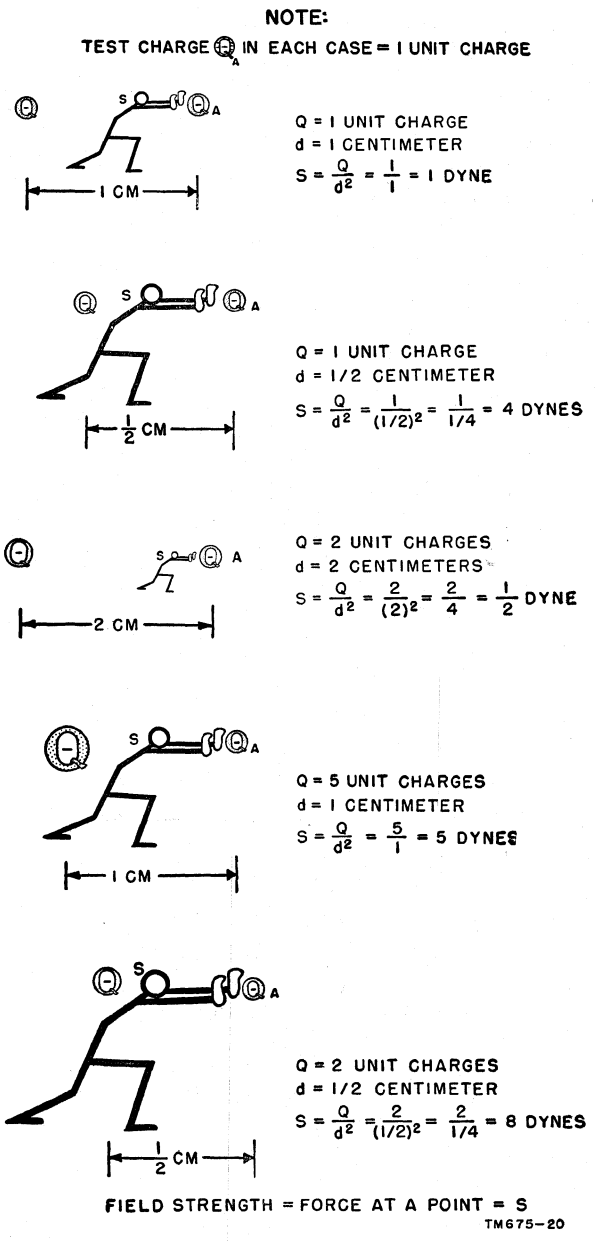

NOTE:
TEST CHARGE Q_A IN EACH CASE = 1 UNIT CHARGE
Q = 1 UNIT CHARGE
d = 1 CENTIMETER
S = Q/d² = 1/1 = 1 DYNE
1 CM
Q = 1 UNIT CHARGE
d = 1/2 CENTIMETER
S = Q/d² = 1/(1/2)² = 1/1/4 = 4 DYNES
1/2 CM
Q = 2 UNIT CHARGES
d = 2 CENTIMETERS
S = Q/d² = 2/(2)² = 2/4 = 1/2 DYNE
2 CM
Q = 5 UNIT CHARGES
d = 1 CENTIMETER
S = Q/d² = 5/1 = 5 DYNES
1 CM
Q = 2 UNIT CHARGES
d = 1/2 CENTIMETER
S = Q/d² = 2/(1/2)² = 2/1/4 = 8 DYNES
1/2 CM
FIELD STRENGTH = FORCE AT A POINT = S
TM675-20

Figure 20. Field strength, S, is force at a point.

12. Field Potential

The difference between field *potential* and field *strength* must be thoroughly understood. Field potential is a measure of *work* and field strength is a measure of *force.* Perhaps this difference can best be understood from the simple equations for electric force and electric potential (TM 11–661). The force equation is $F = Q1Q2/d^2$ and the potential equation is $P = F \times d$. Bearing in mind that potential P equals work W, the potential equation can be written $P = W = F \times d$. As previously explained (par. 11), the force at a point—field strength S—is determined by maintaining $Q2$ of the force equation at unit value (equal to 1);

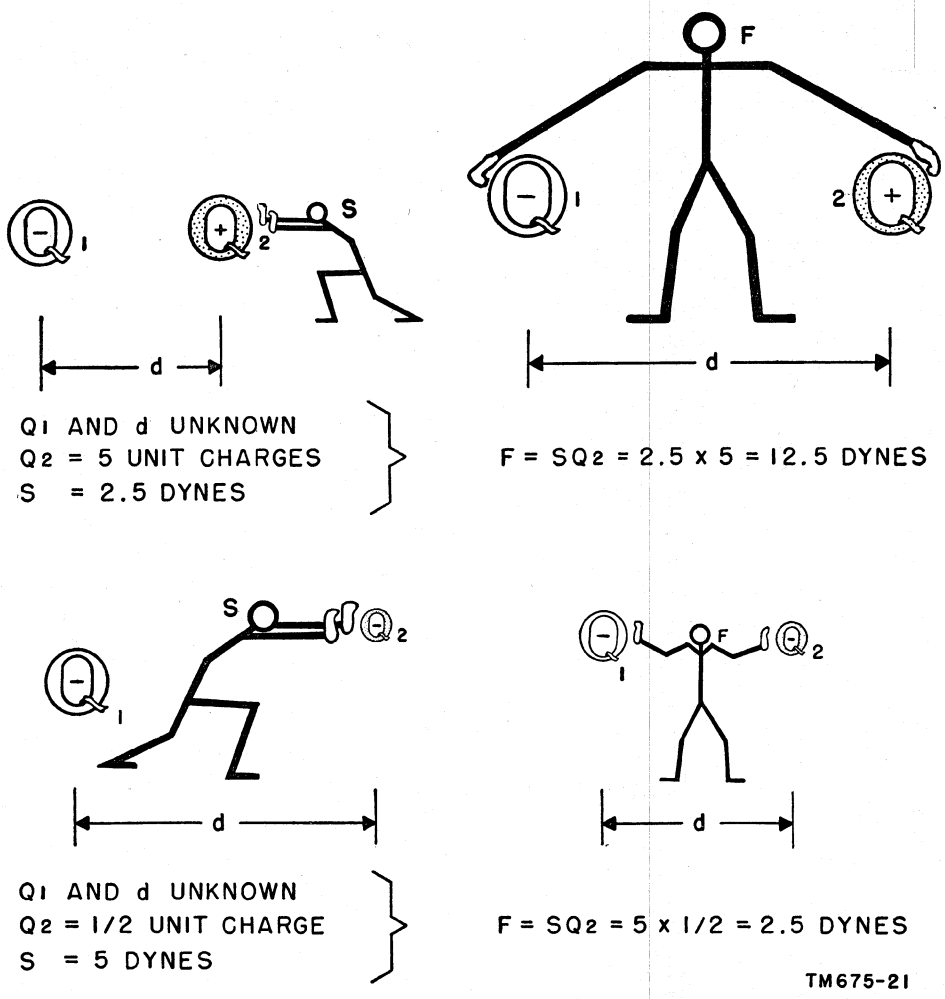

F
S
d
Q1 AND d UNKNOWN
Q2 = 5 UNIT CHARGES
S = 2.5 DYNES
F = SQ2 = 2.5 x 5 = 12.5 DYNES
Q1 AND d UNKNOWN
Q2 = 1/2 UNIT CHARGE
S = 5 DYNES
F = SQ2 = 5 x 1/2 = 2.5 DYNES
TM675-21

Figure 21. Total force can be calculated when field strength, S, and quantity of inserted charge, Q2, are known.

therefore $S = Q1/d^2$. Substituting for F in the potential equation its equal $Q1/d^2$ from the field strength equation, the potential equation becomes:

$$P = W = Q1/d^2 \text{ x } d$$

then, $P = W = Q1/d^2 \text{ x } d = Q1/d$

A direct comparison of the two equations

Field Strength $S = F = Q1/d^2$

Field Potential $P = W = Q1/d$

shows that (1) field strength is a measure of *force,* but field potential is a measure of *work;* (2) both field strength and field potential vary directly with the quantity of charge $Q1$; and (3) field strength varies inversely as the *square* of the distance d of the point from the charge, *but field potential varies inversely as the distance d.*

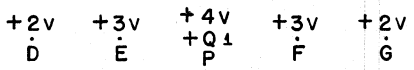
13. Effect of Nearby Charges on Potential

An electric charge and its field are inseparable. For this reason, the potential at a point is not only a function of the quantity of charge (if any) at that point, but is also a function of the kind and quantity of other charges at nearby points.

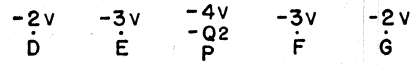
a. Effect of Individual Charge. Assume, for example, that with a quantity of plus charge $Q1$ at point P in A, figure 22, the potential of P is 4 volts. Suppose further that, instead of the quantity of plus charge $Q1$, an equal quantity of negative charge $Q2$ is at point P (B, fig. 22) and that the potential of P is —4 volts. Considering only the field lines between D and G,

values representing the potentials at a few points are shown in A and B, figure 22. These individual potentials are a representation of the field when it is considered that all other charges are too far away to produce any noticeable effect.

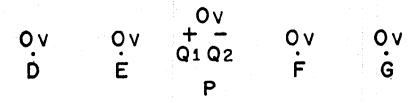
b. EFFECT OF SUPERIMPOSITION. In C, figure 22, the negative charge $Q2$ is superimposed on positive charge $Q1$ and the potential at P and at all other points is zero. This is true because the superimposed equal and unlike charges produce everywhere a neutral condition. Note that C, figure 22, is the equivalent of superimposing A of figure 22 on B and the potentials at points in C are obtained by *algebraic addition* of the potentials at corresponding points in A and B.



A. POTENTIALS ABOUT PLUS CHARGE Q1



B. POTENTIALS ABOUT MINUS CHARGE Q2



C. ZERO POTENTIAL ABOUT SUPERIMPOSED EQUAL AND OPPOSITE CHARGES Q1 AND Q2

TM 675-22

Figure 22. Potential at a point is affected by all nearby charges.

c. EFFECT OF SEPARATED CHARGES. In A, figure 23, conditions are identical to those in A, figure 22, but in B, figure 23, the negative charge $Q2$ is at point F, instead of point P. When the two charges $Q1$ and $Q2$ are placed as shown in C, figure 23, the potential at P is *reduced* from plus 4 volts to plus 1 volt, because of the superimposition at P of a field, due to $Q2$, which has a negative potential of minus 3 volts at P. Similarly, the potential at F is *raised* from minus 4 volts to minus 1 volt; that is, because of the nearby positive charge $Q1$ the effect of the negative charge $Q2$ is reduced. C, figure 23, is the equivalent of superimposing A and B, and the potentials in C are obtained by algebraic addition of the potentials at corresponding points in A and B.

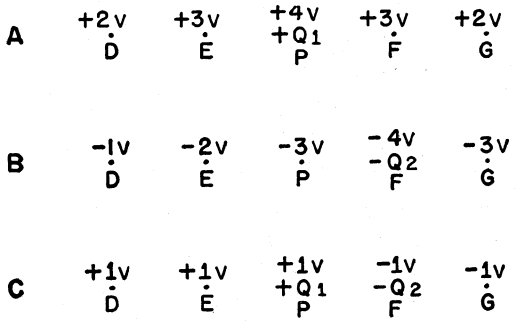
d. POTENTIAL DIFFERENCE AND POTENTIAL. The potential difference between points P and F in C, figure 23, is—

$$PD = P - F = +1 - (-1) = 2 \text{ volts}$$

Thus, the potential at a point is the *algebraic sum* of the individual potentials at that point, but the *potential difference* between two points is equal to the *algebraic difference* between the potentials of the two points. It should be noted that the potential at a point is not clearly defined unless the polarity is indicated, but potential difference is never given polarity. Potential is a *vector* quantity (it has both magnitude and direction) and potential difference is a *scalar* quantity (it has magnitude only). The student has probably noticed that points P, E, and D (C, fig. 23) are at the same positive potential, and points F and G are at the same negative potential. Points at the same potential are called equipotential points and are discussed in the following paragraph.

14. Equipotential Lines

As previously explained, lines of force are drawn to describe the field of force about an

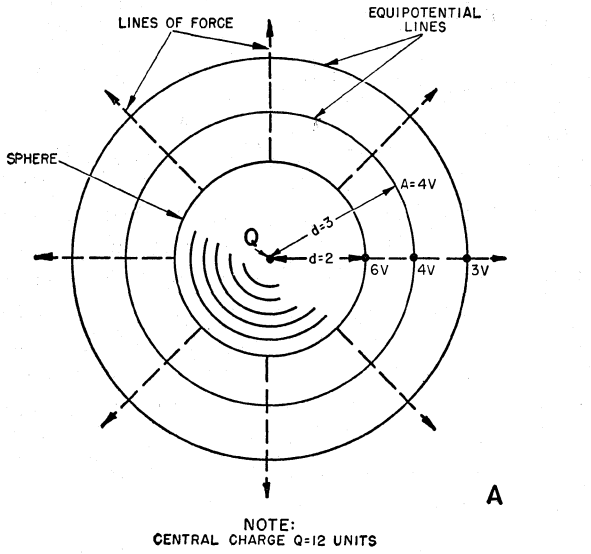

A +2v +3v +4v +3v +2v
D E +Q1 F G
P
B -1v -2v -3v -4v -3v
D E P -Q2 G
F
C +1v +1v +1v -1v -1v
D E +Q1 -Q2 G
P F
AT P:
POTENTIAL DUE TO Q1 = +4
POTENTIAL DUE TO Q2 = -3
POTENTIAL AT P = +1v
AT F:
POTENTIAL DUE TO Q1 = +3v
POTENTIAL DUE TO Q2 = -4v
POTENTIAL AT F = -1v
POTENTIAL DIFFERENCE PDP-F = +1-(-1) = 2v

TM 675-23

Figure 23. Nearby charges must be considered in determining potentials and potential differences.

electric charge. Lines may also be drawn to describe the field of potential about electric charges. Such lines are called equipotential lines and they (usually) represent *surfaces* along any one of which the potential is constant.

a. SPHERICAL SURFACES. Consider that a charge Q of 12 units is at the center of a sphere 2 units in radius (A, fig. 24). The potential of the sphere is 6 units or, assuming practical units, 6 volts. At point A, 3 units of distance from the central charge, the potential is $Q/d = 12/3 = 4$ volts. Now, if this equation is used to determine the potential at every point 3 units of distance from the central charge, it will be found that the potential at each and every such point is 4 volts. *Together* these equipotential points form a spherical surface about the charge Q. Thus, although drawn as circles in the plane of the paper, the equipotential lines in A, figure 24, actually represent concentric spheres about charge Q.



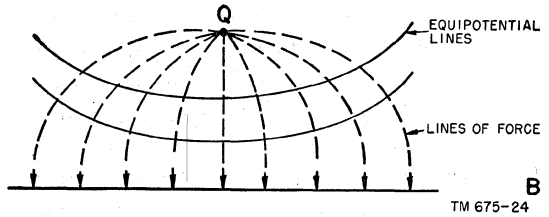


Figure 24. Equipotential lines about a sphere, A, and between a point charged and a flat surface, B.

b. FORCE AND EQUIPOTENTIAL LINES. Some of the equipotential lines about a point charge close to a flat plate are represented by solid lines in B, figure 24, and a portion of the field of force is represented by dashed lines. Notice that at every point of intersection, lines of force and equipotential lines are mutually perpendicular. This means that *there is no component of force along an equipotential surface.*

15. Conductors and Insulators

The atomic structure of certain materials is such that their electrons move freely, but in a haphazard manner, from atom to atom. Other materials do not permit such a haphazard movement of electrons within them and, hence, do not have free electrons. When placed in an electric field, the materials containing large numbers of free electrons permit these electrons to flow readily through them, and are called conductors. Owing to the closeness with which their electrons are bound to the atoms, the other materials do not permit such a flow of electrons, and are called nonconductors, insulators, and dielectrics. The ability to conduct electrons varies in different materials; consequently, there are good and poor conductors, and good and poor insulators. For detailed information on conductors and insulators refer to TM 11–661.

16. Simple Conduction

When a conductor is connected directly across two points between which there is a potential difference, the electric field between the two points necessarily extends through the conductor, and the electrons in the conductor are urged from points of lower to points of higher potentials. A simple example of this concept is shown in figure 25. Assume that chemical action, represented by the dashed line arrows within the battery, removes electrons from plate 1 so that it becomes positively charged and, at the same time, adds electrons to plate 2, so that it becomes negatively charged. This is a separation of electric charges, and an electric field now exists between the two plates; the excess charge on each plate possesses potential energy which is urging it *through* the battery toward the opposite plate. The positive charges on plate 1 cannot move because they are bound to the

atoms in the metallic conductor comprising that plate. The excess electrons on plate 2 possess a potential energy which is trying to force them through the battery, as indicated by the *solid* arrows, but the chemical action of the battery prevents this movement of electrons within the battery.

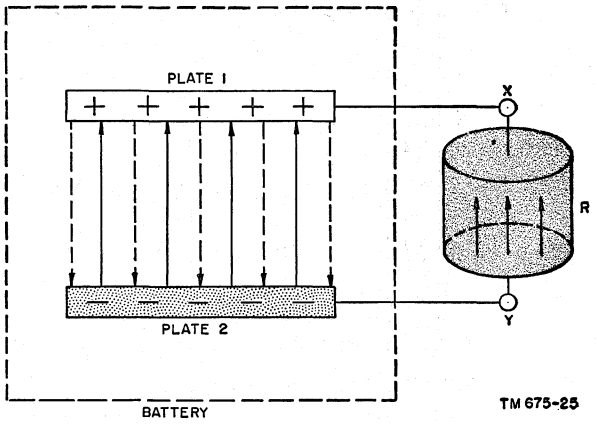


Figure 25. Simplified concept of conduction.

a. SIMPLIFIED EXPLANATION. Conductor R contains electrons that *are* free to move under the influence of an electric field. When conductor R is connected between the two plates, point X is connected directly to plate 1 and assumes the potential of 1; point Y is similarly connected to plate 2 and assumes the potential of 2. Thus, an electric field extends through R as indicated by the solid arrows through R. (For convenience, these lines of force are drawn to show the direction in which the field will urge *electrons*.) The plus charges in conductor R are bound to the atoms and, hence, cannot move under the influence of this electric field. Many negative charges (electrons) in the conductor *are free to move*, and, under the influence of the electric field, *do move* in the direction indicated by the solid arrows. Note that the instant conductor R is connected, *every* electron in the conductor comes under the influence of the field and each electron is urged from points of lower potential (from all points below X) toward points of higher potential at X. As free electrons move upward through R, the potential of X is lowered by the flow of electrons to it and, at the same time, the potential of Y is raised by the flow of electrons from it. However, electrons at X are attracted to points of higher potential on plate 1 and other electrons are drawn from the lower potential at plate 2 toward the increased potential at Y. This drift of electrons, between the point of lowest potential and the point of highest potential at plate 1, tends to reduce the potential difference between plates 1 and 2. However, the chemical action of the battery maintains the potential difference between plates 1 and 2. Thus, there is an electric current through the *conductor and battery* as long as this potential difference is maintained.

b. NEED FOR MORE PRECISE EXPLANATION. When an electric generator and its load are directly connected (fig. 25), the simple explanation given above can be used to explain the transmission of energy from the generator to the load. However, when the generator and the load are separated by appreciable distances, it is necessary to include an explanation of how the *electric field* within the generator is transmitted over these distances. Such explanations must be made in terms of electric and magnetic fields—electromagnetic waves.

17. Electromagnetic Field

It can be shown that whenever an electric field is in motion, there comes into being a force, which is not present while the electric field is at rest. This additional force is called a magnetic force and the space in which it is present is called a magnetic field. In this text a magnetic field is considered as an attribute or property of an electric field in motion. A moving electric field and its associated magnetic field always exist together, and the space in which the two fields are coexistent is called an electromagnetic field.

a. FIELD ABOUT ELECTRON IN MOTION.

(1) *Electric field.* In figure 26, let it be assumed that the electron is moving through space at a constant speed and that its direction of motion is away from the observer. Since the electron and its field are inseparable, the electric field is necessarily moving in the same direction as the electron; i.e., away from the observer. The broken arrow lines of force do *not* represent the direction in which the electric field is in motion; these lines of force repre-

sent only the direction of the field *force*. Considering that the electron motion is uniform, the electric lines of force can be drawn as straight lines.

(2) *Magnetic field.* The magnetic field is represented by concentric circular lines of force with arrowheads indicating the *direction of the magnetic force* about the electron. The magnetic field is due to the motion of the electric field and, consequently, the magnetic field must also be moving away from the observer. Thus, whenever an electron is moving away from an observer, its electric and magnetic fields also move away from the observer and the direction of the resultant circular magnetic field is counterclockwise.

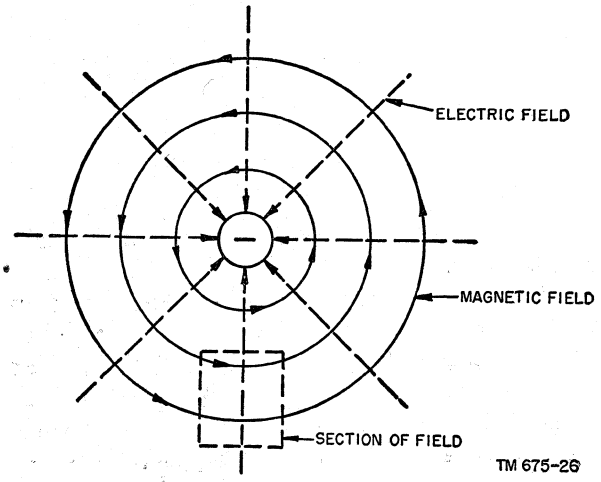


Figure 26. Electromagnetic field about an electron moving away from an observer.

(3) *Fundamental law. At every point in an electromagnetic field the electric field is at right angles to the magnetic field. Also, the direction of the force in each field is at right angles to the direction in which the fields are in motion.* This relationship is usually expressed by saying that the electric and magnetic fields are perpendicular to each other and to the direction of motion.

b. CONSIDERATION OF FIELD ONLY. In dealing with electromagnetic waves, the electromagnetic field must usually be considered without reference to the electron (or other charge) displacement that produced it. For example, if the section of the electromagnetic field in figure 26 is redrawn, as shown in figure 27, so that it can be considered without reference to the electron, the following rule is applicable: A moving electric field and its associated magnetic field are coexistent and so related that, if they are moving from an observer with the electric lines pointed upward, the magnetic lines point to the right.

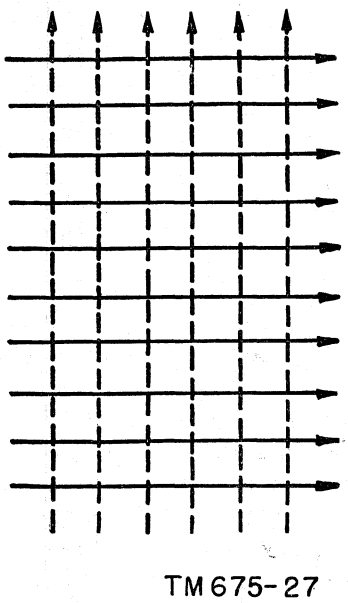


Figure 27. Electromagnetic field moving away from observer.

18. Stationary Magnetic Field

Consider the flow of current through a conductor. First assume that a *uniform* current is flowing from left to right in the conductor shown in figure 28. The solid arrows in this figure then represent the electric field due to the stationary protons (positive charges) within the wire, and the broken arrows represent the electric field due to the electrons. The short arrows pointing to the right indicate that the electron field is moving to the right. Despite this movement of the electron field to the right, there is no *resultant electric field* about the wire, because the two electric fields cancel each other at all points. However, the electron field is an *electric field in motion* and produces about the wire a circular magnetic field. Immediately above the wire, the direction of this magnetic

field is into the paper, and immediately below the wire its direction is out of the paper, as indicated by the conventional circled crosses and dots. Since the current is constant, an observer, looking along the wire in the direction of the electron flow, would see only a *stationary* circular magnetic field about the wire, as shown in B, figure 28. If an electron were at rest at point *P* in the stationary magnetic field, no force would be exerted on it because (1) there is no resultant electric field about the wire, and (2) a magnetic field has no effect on an electric charge, unless there is relative *motion* between the charge and the magnetic field.

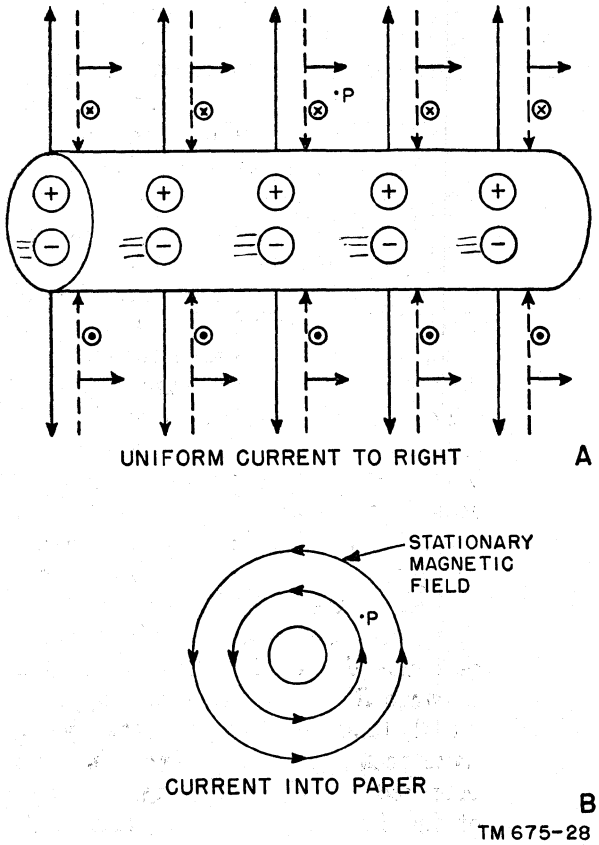


Figure 28. A uniform current produces no electric field, A, but the moving electric field produces a magnetic field, B.

19. Electromagnetic Pulse

For simplicity, A, figure 28, is redrawn in A, figure 29, to show a smaller number of the moving electrons; the positive field is omitted, and *the electric lines of force indicate the direction in which an electron is urged.* Assume that the electron motion to the right is uniform, so that the moving electric field can be considered as made up of straight lines of force. As in the preceding figure, the stationary magnetic field is represented by the circled crosses and dots. The short arrows to the right indicate the direction in which the electrons and their electric fields are in motion.

a. EFFECT OF ACCELERATION. Suppose now, that the current is increasing to a new and higher uniform value. While the current is increasing, the velocity of the electrons is also increasing. Consider that the electrons in A, figure 29 have reached the positions shown in B, figure 29 at the instant that the current and, hence, the electron velocity again become constant. Because the electric field possesses inertia, all parts of this field did *not* move simultaneously to the right with each electron as its velocity increased. The portion of each line of force immediately above each electron has very nearly kept up with the electron motion and is, therefore, adjacent to its electron. However, the portion of each field line between horizontal reference lines *A* and *B* has not kept up with its electron and is now distorted. The electric force in this portion is now to the *left*—i.e., *opposite* to the direction of electron acceleration. The portions of electric field lines between horizontal lines *A* and *Z* are still in their original positions; that is, owing to their inertia the electric field lines between *A* and infinity have not yet moved.

b. EFFECT OF DISTORTED FIELD ON ELECTRONS. An electron in the distorted portion of the electric field would be urged to the *left;* i.e., it would be urged in a direction opposite to that of the electron flow in the conductor. (This phenomenon forms the basis for self and mutual inductance which will be explained later.)

c. PRODUCTION OF PULSE. The electric lines of force in *B* must straighten out and become perpendicular to the surface of their individual electrons. In order to bring about this straightening of the electric lines, the distorted portions must move *outward* from the wire. These distorted portions constitute a moving electric field which produces an accompanying magnetic field. This *electromagnetic* field moves outward from the conductor with the velocity of light. It is called an electromagnetic pulse.

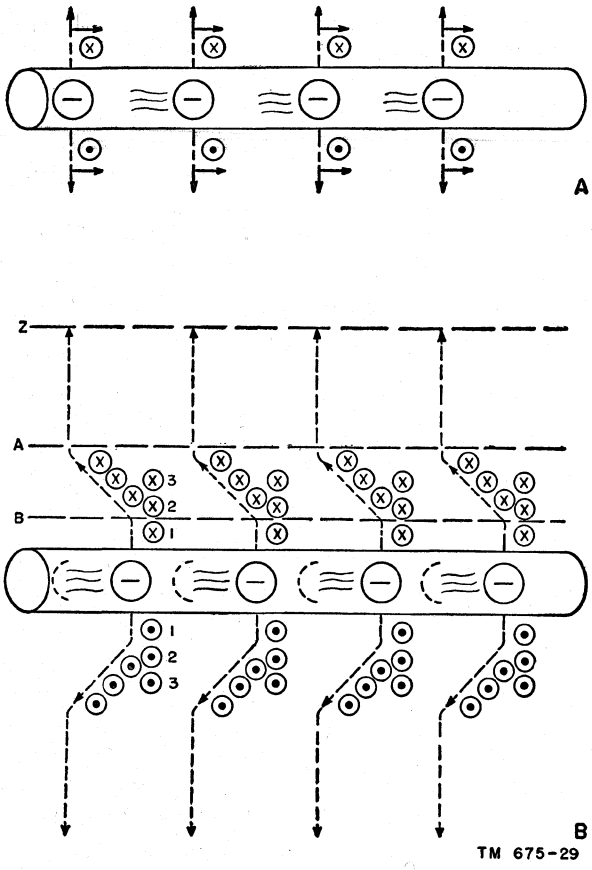


Figure 29. Production of electromagnetic pulse and growth of magnetic field.

20. Moving Magnetic Field

a. GROWTH OF MAGNETIC FIELD. As the electromagnetic pulse moves outward (B, fig. 29), it adds energy to the space through which it moves; that is, in moving outward the pulse carries and leaves behind it the energy which will comprise the stronger stationary magnetic field about the higher uniform current through the conductor. This concept is shown within the limitations of a simple drawing (B, fig. 29). Consider the instant that the distorted portion of the electric field is in the position shown. A stationary circular magnetic line, represented by 1, *has been* established close to the conductor; a second magnetic line, 2, *is being* established, and line 3 *will be* established as the distorted portion moves out from the conductor. At the same time that the magnetic field is being built up about the conductor, the distorted electric field is diminishing and the resultant electric field is becoming zero. This is indicated by the straight electric lines between the conductor and horizontal line B. While the electron velocity (i.e., the current) is *changing*, the field about the conductor is quite complex. For simplicity, it has become customary to neglect the electromagnetic pulse and consider only the changing magnetic field, except when dealing with circuits designed to produce a maximum of radiation.

b. DECREASE OF MAGNETIC FIELD. Now consider the electromagnetic pulse produced when the current and, hence, the velocity of the moving electrons are reduced. For this study, assume that the electron motion in the conductor is uniform and to the right, as shown in A, figure 30. Under this condition, the lines of the electric field (drawn to indicate the direction in which an electron would be urged) are straight and there is a stationary magnetic field about the conductor.

(1) *Effect of acceleration.* Suppose that the current is suddenly reduced to a lower uniform value. At the moment the current is reduced, the electric lines in *A* are moving at a constant speed to the right and their inertia tends to keep them moving at the *same* velocity; therefore, these electric lines do not instantly change their speed of travel to the lower speed of the electron. In consequence of this, at the instant that the electron velocity has again become uniform (B, fig. 30), the portions of the field lines between reference lines *B* and *C* have moved farther to the right than the portions of the lines between the conductor and reference line *B*. In other words, lines between *B* and *C* have moved approximately to the positions that they would have occupied if the current had not been reduced. Note that the direction of the distorted lines and, hence, the electric force between *A* and *B* are now to the right; i.e., the distorted portion will urge electrons in the *same direction* that the conductor current is flowing:

(2) *Reduction of magnetic field.* As in the case of the increasing current previously discussed, the distorted portions of the electric lines move *outward* from the conductor. In this case, how-

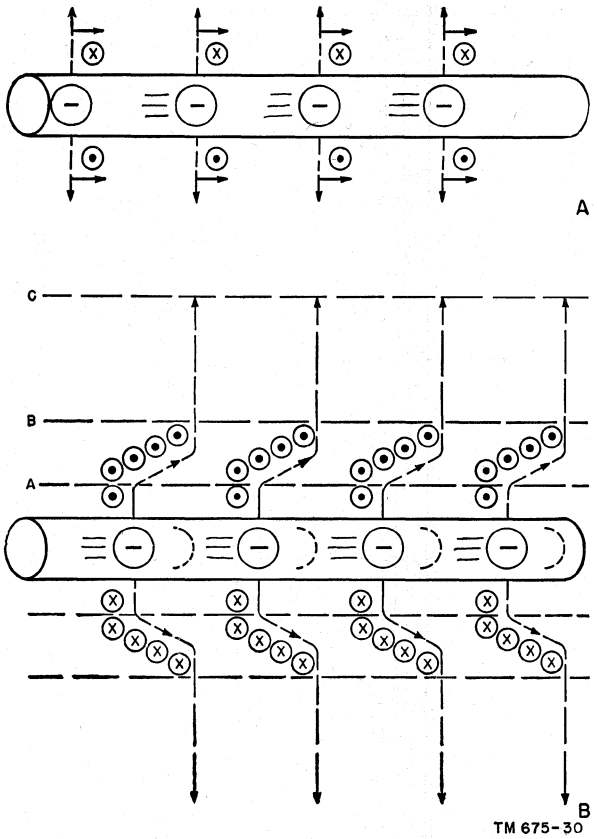


Figure 30. Production of electromagnetic pulse and reduction of magnetic field.

ever, the moving electric lines are pointed to the right and their associated magnetic field lines point *out* of the paper. *In moving outward, this pulse cuts through the magnetic field already existing about the wire.* Since the magnetic component of the pulse itself is *opposite* to that of the existing magnetic field, the existing magnetic field is *weakened.* Note that the *direction* of the existing magnetic force is *not* changed, but the resultant stationary magnetic field becomes weaker than that which existed before the current was reduced.

c. CHANGING MAGNETIC FIELD. As previously mentioned, this complex action may be considered in terms of the magnetic field about the wire (TM's 11–661 and 11–681). In the case of an increasing current, the magnetic field is said to be expanding and, therefore, its lines are moving out from the wire; in the case of a decreasing current, the magnetic field is considered to be collapsing, so that its lines are moving toward the wire. It has been stated that a moving electric field and its associated magnetic field are always coexistent. The converse is also considered as a basic phenomenon; that is, a moving magnetic field and its associated electric field are always coexistent.

21. Self-Inductance

Although usually explained in terms of the changing magnetic field (TM's 11–661 and 11–681), self-inductance can be explained simply in terms of the moving electric field about the charges in motion. First consider the electron fields when the electron velocity is increased in conductor *C* of A, fig. 31. Before the switch *SW* is closed, the current is 0. When the switch is closed, the current would change instantly from 0 to its maximum value of $I = E/R$, if its *change* were not opposed by the self-inductance of the conductor.

a. OPPOSITION TO CURRENT CHANGE (fig. 31). At the moment the switch, *SW*, is closed, the applied voltage, *E*, produces an electric field through the conductor, *C*, and this field is an applied force f_1 which urges the free electrons toward points of higher potential (upward in fig. 31). The current tries to rise instantly to its maximum value of E/R. However, as the electrons start their net movement upward, their velocity is *changing* while increasing from zero. The elemental field of each electron, therefore, becomes distorted, as explained in connection with figure 29. Assume that two individual electrons, 1 and 2, are in the positions shown in B, fig. 31. When these electrons start upward under the influence of the applied voltage, their individual electric fields become distorted, so that the force in the distorted portion of the field of 1 is in a direction *opposite* to that of the applied field. It is apparent that this distorted portion of the field about electron 1 is opposing the upward movement of electron 2. The field of *every* moving electron in the wire is similarly distorted and, consequently, the upward movement of every electron is opposed. Thus, an opposing force is exerted, not only on electron 2, but at every point in the conductor. This opposing force is called an induced force f_2 and it prevents the current from instantly

reaching its maximum value equal to E/R. After the current reaches its E/R value, the electron velocity is uniform and the elemental electron fields are no longer distorted. Under this condition, there is no induced force to oppose the electron flow, and the current is limited only by the resistance of the circuit. The opposing effect of inductance is present only when the current is *changing,* and the induced force is always in such a direction that it opposes the *change.* If the applied voltage E is reduced, the current must fall to a lower value. However, as explained in connection with figure 30, the distorted portions of the electron field are now reversed (C, fig. 31). Therefore, the induced force and the applied force are now in the *same* direction. Note that by now aiding the applied force the induced force is opposing the current *change.*

b. INDUCED FORCES GENERATE VOLTAGE. If only the induced force were present through the conductor in B, figure 31, it would urge electrons from X toward Y, thus generating a potential difference between the conductor ends, end X becoming positive, end Y becoming negative. The induced force in C, however, would urge electrons from Y toward X, causing X to become negative and Y to become positive. These *generated* voltages, due to an induced force, are called induced voltages.

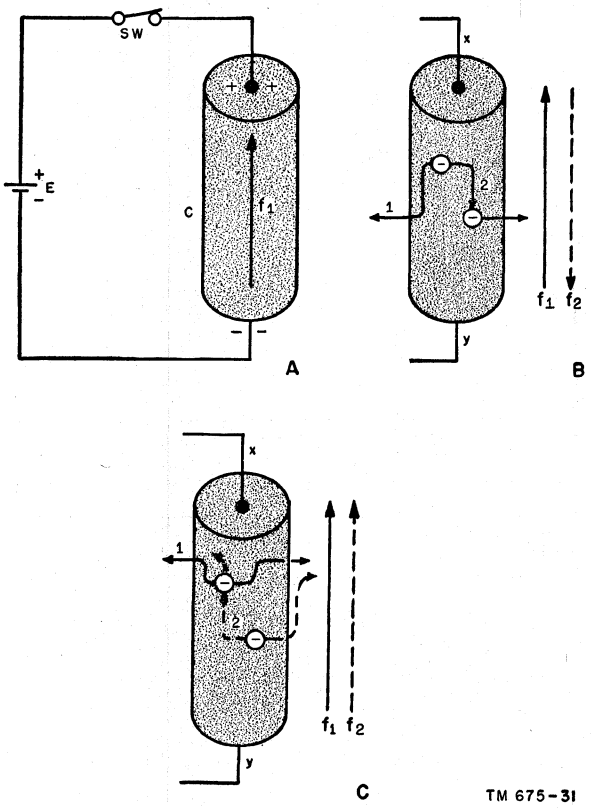


Figure 31. Self-induction in terms of electric fields.

22. Capacitance

The amount of charge on a conductor is not only a function of its physical characteristics, but is also proportional to the potential V. In electrostatics, the quantity of charge Q that can be stored on a set of parallel plates is expressed by the equation

$$Q = KAV/4\pi S$$

where Q is the total charge on the plates, A the area of the plates, K the dielectric constant of the medium between the plates, S the separation of the plates, 4π a constant, and V the potential difference. The charge on *each* plate is proportional to the potential difference V between the plates.

a. DEFINITION. *Capacitance is the ratio of the quantity of electricity to its potential V, or* $C = Q/V$.

(1) *Concept.* Capacitance is a useful term because it indicates how much charge is stored on a conductor with a given potential difference. The *farad* is the practical unit of capacitance; a *conductor* has a capacitance of 1 farad when its *potential* is changed 1 volt by 1 coulomb of electricity; a capacitor has a capacitance of 1 farad when the *potential difference* between its plates is changed 1 volt by a transfer of 1 coulomb of electricity from one set of plates to the other. The capacitance of capacitors is generally much less than 1 farad and is usually measured in μf (microfarads) or in $\mu\mu$f (micromicrofarads).

(2) *Practical application.* Since $C = Q/V$, a capacitor or conductor has a large capacitance if its potential is raised a *small* amount by a *large* quantity of electricity. For example, if the transfer of one microcoulomb of electricity from one set of plates to the other in

a capacitor changes its potential by 1 volt,

$$C = Q/V = \frac{1 \times 10^{-6}}{1} = 1 \times 10^{-6} \text{ farad} = 1 \ \mu\text{f}$$

If the similar transfer of four times as much electricity (i.e., 4 microcoulombs) changes the potential of a second condenser by 1 volt, the capacitance of the second capacitor is four times greater than that of the first,

$$C = Q/V = 4 \times 10^{-6}/1 = 4 \times 10^{-6} \text{ farad}$$

$$C = 4 \ \mu\text{f}$$

b. EFFECT OF PLATE SEPARATION. Since 4π is a constant, if K and A are also held constant, the formula $Q = KAV/4\pi S$ can be simplified to $Q = V/S$. If Q is maintained constant, the potential V can be found for various values of S,

$$V = QS$$

Consider that a conductor has 1 unit of charge and that its separation (i.e., distance) from the earth is 1 unit of distance. Then,

$$V = QS = 1 \times 1 = 1 \text{ volt}$$

The potential of the conductor will increase as S is increased. With S increased from 1 to 2 units of distance,

$$V = QS = 1 \times 2 = 2 \text{ volts}$$

c. CHANGE OF C AS S IS VARIED. Since V changes as S is changed, it follows that C must also vary with changes in S because

$$C = Q/V = \frac{Q}{QS} = \frac{1}{S}$$

Consider that a wire 2 inches long and .05 inch in diameter is parallel with and .2 inch above the ground. Under this condition, assume that the wire capacitance to ground is 1 $\mu\mu$f. A charge Q of 1×10^{-12} coulomb (1 micromicrocoulomb) changes the potential difference between the wire and ground by 1 volt, since

$$C = Q/V$$

and

$$V = Q/C$$

$$V = \frac{1 \times 10^{-12}}{1 \times 10^{-12}}$$

$$V = 1 \text{ volt}$$

Assuming that the wire is at a positive potential with respect to ground and that the charge Q remains constant, the potential increases to 2 volts when S is doubled by raising the wire to a height of .4 inch. Although the wire retains the *same quantity* of electricity at the greater distance from ground, its capacitance is halved, for

$$C = Q/V = \frac{1}{S}$$

$$C = \frac{1 \times 10^{-12}}{2}$$

$$C = 5 \times 10^{-13} \text{ farad}$$

$$C = .5 \ \mu\mu\text{f}$$

The graph in figure 32 shows that the variation of the capacitance C and the potential V as S is increased by raising the wire from .1 inch above the ground to a height of 20 inches. It might be well to note that the graph values of C are not accurate values, since they are based on an assumed value of 1 $\mu\mu$f at a separation S of .2 inch. The only purpose of the graph is to show in a general way the decrease of capacitance C as spacing S is increased.

d. INFINITY AND ZERO. It is reasonable to assume that as the separation S is increased to a distance greater than can be represented by any number, no matter how great the number, the potential V will also increase to a value which is also too great to be represented by a number. A value which is so great that it is always *greater* than any number that can be written is said to be infinite; an infinite value is represented by the symbol ∞.

(1) *Zero capacitance.* Since the capacitance is inversely proportional to the potential, if V is made infinite by increasing S to an infinite value, the capacitance under this condition is said to be 0 for

$$C = Q/V = 1/\infty = 0.$$

The concept that 1 (or any number) divided by an infinite value—i.e., by ∞ — is always equal to 0 can be understood from an examination of figure 32. With increasing numerical values of S, C becomes smaller and smaller and approaches a value of 0. Consequently, if S is increased to a value greater than the largest number that can be written, it is reasonable to assume that C does become 0 in accordance with the mathematical concept that $1/\infty = 0$.

(2) *Practical considerations.* Of course, it is impossible to move a wire to an infinite distance. However, scientists have concluded that the velocity at

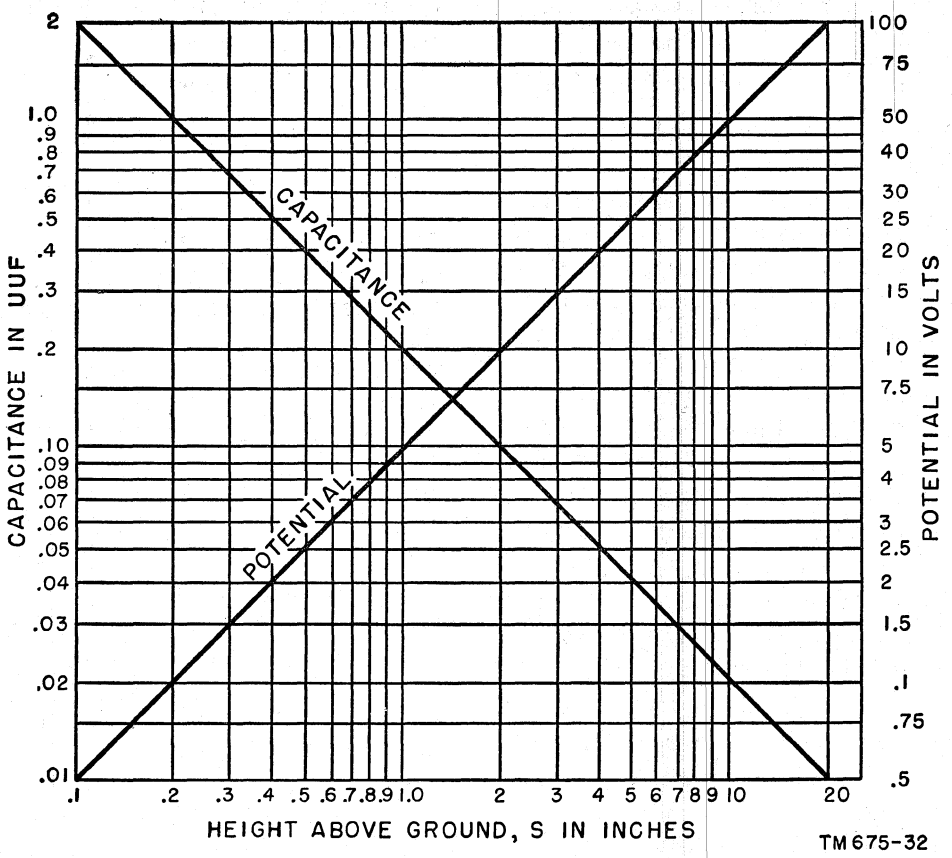


Figure 32. Graph of variation of capacitance and voltage as wire is raised above ground.

which electromagnetic waves travel through free space is equal to the velocity of light. If we assume that a wire is in free space, its capacitance to ground is zero and the velocity at which an electrical disturbance travels along the wire is thought to be equal to the velocity of light in free space. The relationships of wire length, wavelength, and frequency are functions of the wire inductance, capacitance, and resistance.

Section III. ELECTRIC TRANSMISSION

23. Application of Basic Concepts

a. ANALYSIS OF LINE ACTION. The action of any transmission line can be analyzed by means of mathematical equations. The advantage of mathematical analysis is, of course, that a single equation may explain a complicated circuit action — precisely, quantitatively. It is frequently and truly said: "One picture is worth a thousand words," and, similarly, "One equation is worth a thousand words and ten pictures!" Because this manual is intended primarily for those who have not had the opportunity to study mathematics, only a few of the most useful equations are included in the text.

b. SIMPLIFICATION. In most transmission-line circuits, a number of electrical effects occur simultaneously to produce the resultant line action. Whenever possible, simplicity is obtained by first considering the line in terms of each electrical effect as though only that effect were present. For example, in explaining the movement of voltage from one end of a line to another, the voltage (i.e., potential) at a point is

considered as being due to the excess charge existing at that point. The *apparent* movement of this excess charge from point to point is then explained by the effect of its field on other charges within the wire. In this manner, reflection and oscillation are first described without reference to capacitance, inductance, and resistance. Also, unless otherwise indicated, it is assumed in this chapter that the wires or lines are in free space; that is, so far removed from earth that their operation is not affected by anything external to them.

24. Movement of Point of Charge

The apparent movement of an excess of one kind of charge from point to point along a wire and, hence, the movement of a *point of charge* or *point of potential* from one end of the wire to the other can be explained in simple terms.

a. THEORETICAL CONDITIONS. Consider that point 0 in A, figure 33, is the near end of a straight wire which extends to infinity; that is, the wire is so long that the far end can never be reached. Also consider that the wire is isolated in free space, so far from the earth that capacitance to ground can be neglected. Before an excess of one kind of charge is produced on its near end at 0, the wire is electrically neutral and every point on the wire is at the same potential. It is impossible to illustrate the distribution of the millions of opposite kinds of electric charge within the wire.

b. EQUIVALENT POSITIVE CHARGE. Although the plus charges within the wire cannot be moved along the line, the *equivalent* of a movement of positive charge in one direction is brought about by the actual movement of negative charge in the opposite direction. Suppose that a portion of the normal negative charge (represented by electron *A*) is in some way moved from point 0 (A, fig. 33). When this is done, as in B, electrical balance no longer exists at point 0, because plus charge X (inclosed in dashed line oval) forms an *excess* positive charge at 0. In other words, the movement of electron *A* from point 0, and the consequent establishment of the positive charge, raise the potential of point 0. A potential difference now exists between 0 and every other point in the wire.

c. FIELD DUE TO CHARGE. Theoretically, the electric field of the positive charge established at 0 extends in *every* direction from 0 to infinity. At the moment the positive charge is established, the resultant electric field within the wire is strongest in the immediate vicinity of 0, and much weaker at points farther along the wire. For example, if the mutual attraction between the plus charge at 0 and electron *B* at point 1 is 1 unit, the mutual attraction between an electron *C* at point 2 (twice the distance 0–1 from 0) and the plus charge at 0 may be less than ¼ unit. Similarly, point 3 is three times the distance 0–1 and, therefore, the attractive force at point 3 may be less than ⅑ unit. This difference in force is roughly indicated by the relative lengths of the arrows on the electrons.

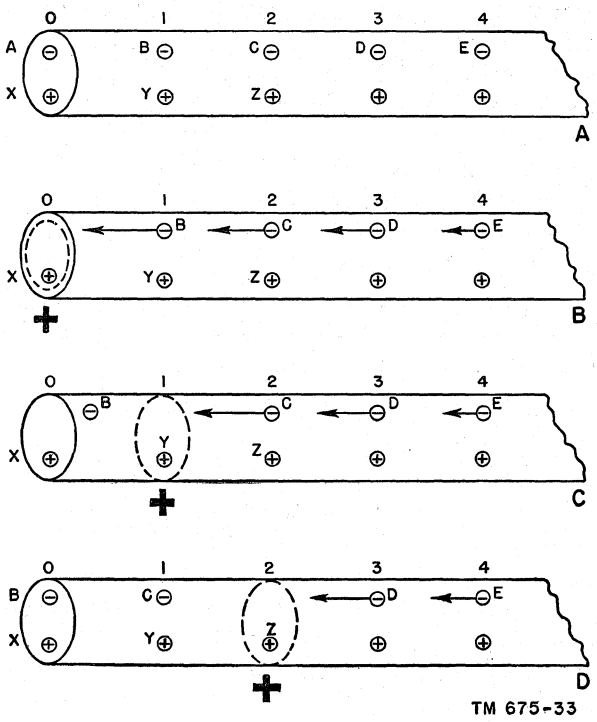


Figure 33. Simplified concept of movement of point of plus charge.

d. CURRENT IN VICINITY OF CHARGE. Consider that when the positive charge is produced at 0, the resulting field urges every electron in the wire toward 0 (B, fig. 33). However, because the greatest force is acting on the electrons which are nearest to the positive charge, the free electrons in this vicinity start moving toward 0 at a greater velocity than the more

distant electrons. This *maximum current to the point of positive potential reduces the potential at 0.* Electrons between 0 and 1 move away from point 1 much more rapidly than the more distant electrons move into point 1; therefore point 1 (C, fig. 33), now becomes positive because of its consequent *momentary lack* of electrons. As shown in C, figure 33, no positive charge has moved, but this current or movement of *negative* charge, *B, out* of point 1 and *into* point *0* is the equivalent of moving the excess of positive charge (dashed line oval) in the reverse direction (i.e., from point 0 to point 1). Consequently, the potential of point 1 is now greater than that at every other point in the line.

e. TERMINOLOGY. For simplicity, it has become customary to say that the charge or voltage moves down the line. It should be noted, however, that *individual* positive charges within the wire have *not* changed their positions; the movement of a group of electrons to the left constitutes a current that has produced an *equivalent* movement of the point of positive charge or potential in the reverse direction (to the right). The potential at point 1 thus becomes essentially the same as it would be if the original positive charge *X* had actually moved along the wire.

f. CONTINUED MOVEMENT OF CHARGE. With the equivalent positive charge moved to point 1 (C, fig. 33), the greatest electric force is now acting on all the electrons between points 1 and 2; therefore, the free electrons in this vicinity now comprise the maximum current and move very rapidly toward the excess positive charge *Y* at point 1. In moving away from positive charge *Z* at point 2, this current is flowing *into* point 1 to neutralize the positive charge there. At the same time, however, this current flows *away* from point 2 and the positive charge *Z* becomes effective as the equivalent positive charge. As indicated in D, figure 33, the equivalent positive charge and, therefore, the *point of positive voltage* has moved to point 2, and will continue to move toward the far end of the line. In this assumed case, the length of the line is infinite and the positive charge would move down the line forever without reaching the far end.

g. SPEED OF TRAVEL. A further examination of figure 33 will show that *no individual electron has traveled any great distance from its original position on the line.* However, the equivalent positive charge—the *point* at which there is an excess of protons, the *point of electrical unbalance*—travels down the line at the speed of light—i.e., at a velocity of 186,000 miles per second. Note that the current in the wire is always maximum in the immediate vicinity of the equivalent positive charge as it moves along the line; that is, *the voltage and current sweep down the line together and in phase.* In actual wire, the velocity is somewhat less than the speed of light in free space, because of the resistance and shunt conductance present in a real line. (The effects of these electrical constants will be considered later).

h. MOVEMENT OF POINT OF NEGATIVE POTENTIAL.

(1) *Field due to charge.* The electrical unbalance caused by an excess of negative charge at point 0 will also travel down a straight wire in much the same manner as the equivalent positive charge. In A, figure 34, the near end of the infinite line is again shown in its neutral condition. In B, figure 34, the near end of the line contains an excess of negative charge (represented by electron A^1). Since point 0 now has an excess number of electrons, its potential is *lower* than that of every other point along the wire and there is a potential difference or voltage between point 0 and every other point along the line. The field of the negative charge thus established at 0 extends in every direction to infinity. The electric force of this negative charge is in the reverse direction to that of the positive charge previously established at this end of the line (B, fig. 33). This means that the electric field, due to the excess negative charge at 0, is forcing every free electron *away* from the near end of the line. This force is greatest on the electrons in the immediate vicinity of the negative charge and, therefore,

a maximum current occurs at this point as the nearest electrons move very rapidly away from 0 and toward point 1.

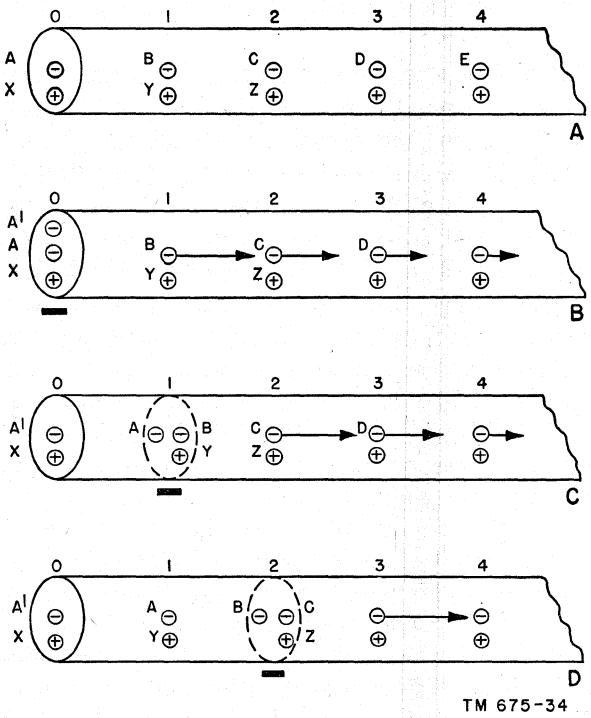


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Figure 34. Simplified concept of movement of point of negative charge.

(2) *Movement of charge.* In this case, negative charge can and does move along the wire from the positive charge *X* toward point 1 to produce a balanced electric condition at the near end of the line. At the same time, the flow of current into point 1 produces there an excess of negative charge (C, fig. 33). In turn, this increased negative charge at point 1 causes the free electrons between 1 and 2 to start moving rapidly toward point 2, and the point of electrical unbalance (inclosed in dashed lines in fig. 34) continues to move down the line in the manner described (C and D, fig. 34). It should again be noted that, although it is said that the negative charges down the line with the speed of light, the actual displacement (i.e., distance of travel) of any *individual* electron is slight.

i. SUMMARY. Whenever an excess of one kind of charge exists at a point, there is an electric potential at that point. Consequently, when the equivalent positive charge is first established on the wire (B, fig. 33), there is a potential difference or voltage between point 0, which contains an excess number of positive charges, and the remainder of the line which, for the moment, contains equal numbers of positive and negative charges. As the negative charges in the line shift successively to the left to neutralize the positive charge, the point of electrical unbalance or positive potential moves down the line with the speed of light. As the point of potential moves along the line, the current is always greatest in its immediate vicinity and the voltage and current are said to travel in phase. Comparison of figures 33 and 34 will show that the movement of a point of negative charge down the line is similar to that of the point of positive charge, except that the current due to the charge is in the reverse direction.

25. Reflection of Charge

Before considering the effects of the inductance, capacitance, and resistance distributed along the length of an actual wire, it is well to discuss here the movement of a charge along a wire which is not infinite in length. If an electric charge (whether plus or minus) is established at either end of a relatively short uncharged line, the point of potential will move toward the opposite end of the wire.

a. POSITIVE CHARGE. As shown in A, figure 35, if a positive charge is first established at the left end of the wire, it will move to the right; however, if the positive charge is first established at the right end of the wire, it will move in the opposite direction, i.e., to the left as indicated in B, figure 35. As previously pointed out, the equivalent of moving a positive charge along a metallic conductor is actually accomplished by moving electrons (current) in the opposite direction. Since the equivalent plus charge in A is moving in the opposite direction to that in B, the current in A is also the reverse of that in B. (Arrows within the wires indicate direction and approximate position of maximum current.)

b. NEGATIVE CHARGE. The negative charge first established at the left end of the wire in C, figure 35, moves to the right, and the equal negative charge first established at the right in D, figure 35, moves to the left. Since the established charges in C and D are negative, *the current is in the same direction as the movement of charge or potential.*

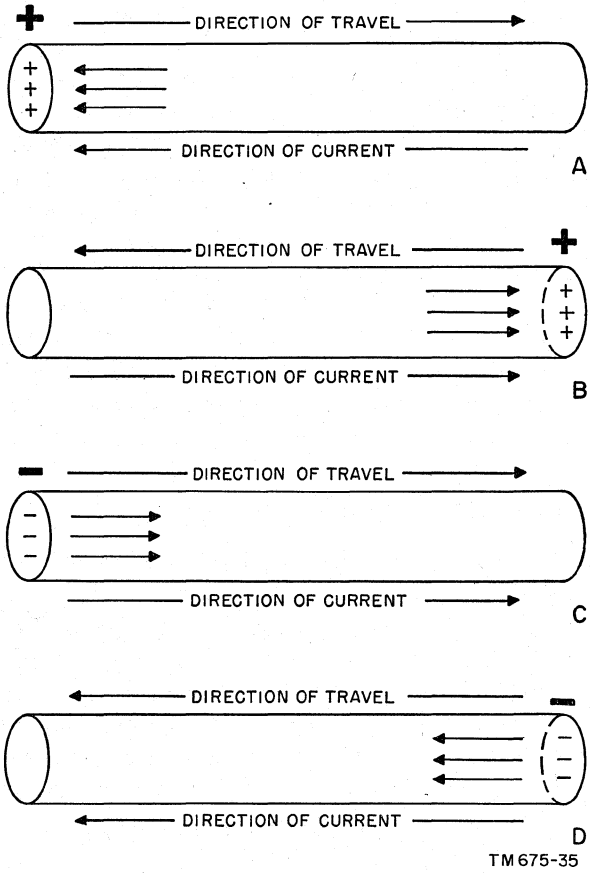

Figure 35. Movement of points of charge on wires of finite length.

c. OSCILLATION. Comparing A and B in figure 35, it is reasonable to assume that when the plus charge first established at the left in A reaches the right end of the line, the electrical condition of the line will be essentially the same as that of the line shown in B. Under this condition, the plus charge will reverse its direction of travel and move back over the line toward its original position at the left. Following this line of reasoning, it is also feasible to assume that upon reaching its original position at the left, the plus charge will again reverse its direction of travel and move toward the right end of the line. Assuming that none of its original energy is radiated from the wire or lost within the wire, the positive charge in A, or the negative charge in C, will continue to move back and forth, or oscillate, along the wire forever.

d. COMPLETE REFLECTION. In this case, the wire is assumed to be a good conductor and the medium surrounding the wire is assumed to be a perfect nonconductor or insulator. Consequently, as the electric charge, positive or negative as the case may be, reaches either end of the wire, it encounters a perfect insulating medium through which it cannot move and therefore it must stop. Since it is assumed that the stopped charge has not lost any energy, its electric field exerts a force on every electron within the wire. This force of the electric field on the electrons now causes a reversal of current within the wire and a consequent movement of the charge back over the wire. Because the reversal of its direction of travel is comparable to the reflection of light from the surface of a mirror, the charge is said to be reflected from the end of a line. The above-described reflection is said to be complete reflection, for all of the energy reaching the end of the line is reflected back over the line; that is, no energy is dissipated in the medium at the end of the line. Whenever the terminating medium absorbs *all* the energy reaching it from the line, *no reflection can take place.* In some cases, only a portion of the energy may be absorbed in the terminating medium and the remainder of the energy is reflected back over the line. The various line terminations and their effects on reflection will be discussed in detail later.

e. POLARITY. A further examination of figure 35 will bring out a very important action which must be thoroughly understood. The equivalent positive charge represents a *point* of positive potential, or voltage, which moves from the left to the right in A. Upon being reflected, however, the point moves from the right to the left, as shown in B. Note that although this potential reverses its *direction of travel,* there is *no reversal of polarity.* In other words, regardless of its direction of travel, the charge is the equivalent of a point of higher potential which is moving back and forth over the line.

f. REVERSAL OF CURRENT WITHOUT REVERSAL OF POLARITY. Comparison of the current flow in A, figure 35 with that of B will show that there *is a reversal of current* when the excess charge or point of potential is reflected from either end of the line. This reflection of a charge (or potential) without reversal of polarity, but with a reversal of current flow, occurs whenever the line is terminated in a high impedance that is the equivalent of a high resistance. The negative charge or potential established at the left in C, figure 35, travels to the right end of the line and then reverses its direction of travel as shown in D, but in traveling back over the line the *negative* potential does not reverse its *polarity*. In this case also, there is a reflection of potential without a reversal of polarity, but with a reversal of current.

26. Capacitance of a Wire

The capacitance of a wire is comprised of its capacitance to ground and to other objects, and its capacitance independent of ground and all other bodies.

a. SELF-CAPACITANCE. Theoretically, if a wire is moved to an infinite distance from the earth, the capacitance of the wire to ground and to other objects becomes zero (par. 22). If it were possible to study a wire under these theoretical conditions, it would be found that a charge could be taken from or added to the wire; that is, the wire has *self*-capacitance. This concept is at first somewhat difficult to understand, because it cannot be visualized directly in terms of the familiar two-plate capacitor.

b. SIMPLIFIED EXPLANATION. Consider that the wire in A, figure 36, is isolated in free space and that oscillations can be stopped at the moment when the opposite ends contain maximum charge. At this instant, slice end sections 1 and 2 from the wire and place them as indicated in B, figure 36. These end sections and the dielectric material about the wire form a very small two-plate capacitor (1–2 in C, figure 36). This *end capacitance* is only a portion of the self-capacitance; *there is capacitance between each section and every other section on the wire.* This idea of *distributed* self-capacitance is shown, within the limitations of a simple drawing, by the dotted line capacitors between points 3 and 4 and 5 and 6 in C, figure 36.

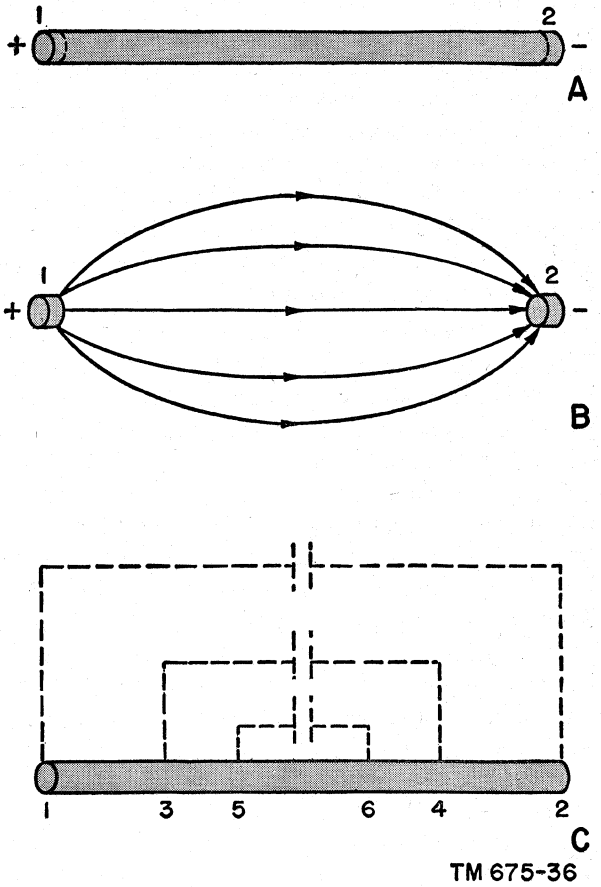

Figure 36. A wire has self-capacitance.

27. Inductance of a Straight Conductor

Every conductor, regardless of its size and shape, possess self-inductance. This self-inductance may be explained either in terms of the magnetic field (TM 11–661) or in terms of the electric field (par. 21). These explanations are equivalent and cause no confusion when it is remembered that the magnetic field is an attribute of the electric field. Explanations in terms of the electric field are sometimes more direct. The important point to remember is that *every* conductor has self-inductance. Thus, the shortest possible piece of wire has self-inductance. Although the *amount* of self-inductance in such a short wire may be extremely small, its *effect* may be considerable at high frequencies.

28. Resonant Wire

The movement of a *point* of potential or charge along a wire and reflection from one end to the other have been considered without reference to the wire inductance, capacitance, or resistance (par. 24). These simplified explanations show that a *point* of charge (or potential) necessarily moves because of the resultant electric field and its effect on charges in the wire. However, the relationship of wire length, frequency, and wavelength can be explained only by including inductance and capacitance in the discussion.

a. COMPARISON OF WIRE AND L-C CIRCUIT. It is possible to compare a length of wire directly with a coil and capacitor circuit. To show practical equivalence, assume that the length of wire in figure 37 has points of charge that are moving back and forth over the wire (par. 25). Now consider the total distributed capacitance of the wire as concentrated in the capacitor formed by the end sections (B, fig. 36). This capacitance corresponds to the lumped capacitance C in figure 37. With a lumped inductance L equal to the distributed inductance of the wire, the L–C circuit of figure 37 can be considered as a practical equivalent of the wire.

b. CIRCUIT ACTION. Refer to figure 37 where the electrical conditions of a wire and its corresponding L–C circuit are represented for 1 cycle of oscillation. Times are given by the time circle T_1 through T_8 (clockwise). The electric and magnetic fields are represented by conventional lines of force. In the wire the *point of maximum positive charge* is represented by circled plus signs (⊕ ⊕) and the *point of maximum negative charge* is represented by circled minus signs (⊖ ⊖). The short arrows indicate the directions in which these *points* of charge are moving. The direction of current in the L–C circuit is indicated by arrowheads on the connecting wires. The potentials of the wire ends and of the capacitor plates are represented by conventional plus and minus signs.

(1) At moment T_1 (fig. 37) the maximum potential difference exists between the ends of the wire, and the points of charge are also at the ends of the wire. At this instant the distributed capacitance of the wire and the capacitor in the L–C circuit are fully charged. Thus, in both circuits all of the energy is in the electric field. Note that at this instant there is no current and, hence, no magnetic field about either circuit. Refer to TM 11–681 for a detailed discussion of parallel resonant circuits.

(2) At moment T_2 the points of charge are moving from the ends of the wire and are closer together. While the points of charge are in motion, there is current in the wire (par. 24); consequently, there is a magnetic field about the wire. Note that the potential difference between the ends of the wire is decreasing, because the points of charge are moving closer together (par. 13). At this same instant the voltage across the capacitor in the L–C circuit is also decreasing, because the capacitor is discharging through the inductance. This discharge current produces a magnetic field about the inductance L.

(3) At moment T_3 the points of charge are together (superimposed) and there is no potential difference between the ends of the wire. At this same instant, the capacitor in the L–C circuit is fully discharged, the capacitor voltage is zero, current through the inductance is maximum, and all of the circuit energy is in the magnetic field.

(4) At moment T_4 the points of charge have passed one another in the wire, and, because these points are no longer superimposed, there is again a potential difference between the wire ends. However, the points of charge have reversed their positions relative to one another and there is a consequent reversal of polarity. Compare T_4 with T_1 and T_2. Note that although the magnetic field is *collapsing*, its direction has not reversed. In the L–C circuit the magnetic field is also collapsing without a reversal of direc-

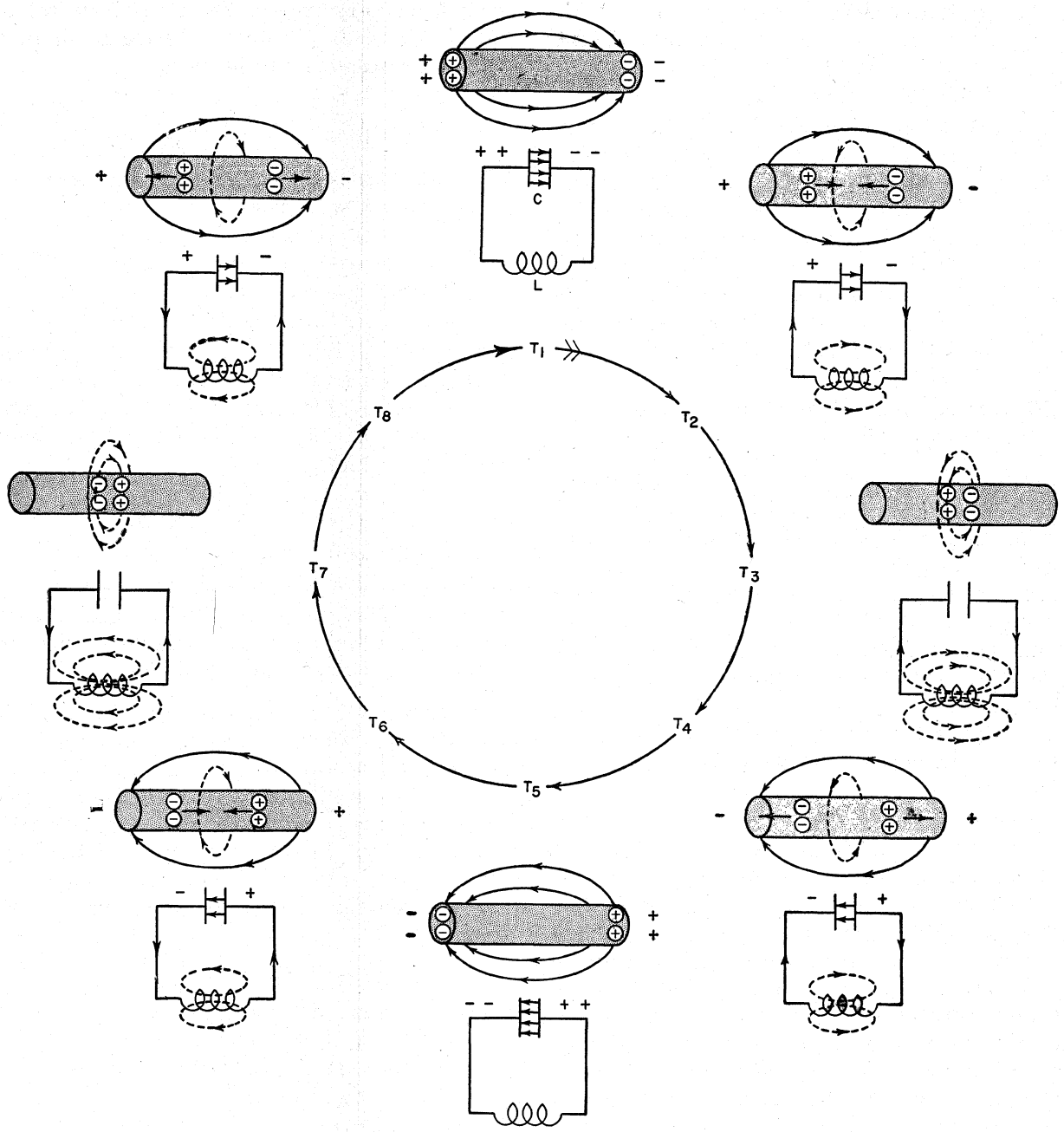

Figure 37. Comparison of oscillations in a wire and in an L–C circuit.

tion. This collapsing field induces a voltage which causes current in the original direction; consequently capacitor C is now charging with reversed polarity.

(5) At moment T_5 the points of charge have reached the ends of the wire, there is maximum potential difference between the wire ends, the current is zero, and the magnetic field is zero. All of the energy is again in the electric field. Conditions in the L–C circuit correspond to those in the wire; that is, the capacitor is fully charged,

the capacitor voltage is maximum, the current is zero, the magnetic field is zero, and all of the circuit energy is in the electric field.

(6) As explained for time T_1 through T_5, the circuit action continues through times T_6, T_7, T_8, and T_1, except that the points of charge move in the reverse direction and capacitor C discharges in the opposite direction through inductance L. One cycle of oscillation is completed when the conditions of T_1 are reestablished.

29. Frequency, Wavelength, and Wire Length

Resonant circuit theory (TM 11–681) shows that the frequency of a resonant L–C circuit is a function of L and C. Comparison of the wire and the L–C circuit (fig. 37) indicates that the frequency of oscillation in a wire is also a function of L and C. In a wire the inductance and capacitance are distributed, instead of being lumped as in the familiar *coil and capacitor* circuit.

a. EQUATION FOR FREQUENCY, F. The equation for frequency in terms of L and C is:

$$F = \tfrac{1}{2\pi}\sqrt{LC}$$

where: L is in henrys
C is in farads
F is in cycles

b. VARIATION OF L, C, AND F WITH WIRE LENGTH. With everything else equal, the distributed inductance and capacitance of a given wire are proportional to the wire length. Also, for a given frequency any combination of L and C that will give the required L–C product ($L \times C$) may be used to form a resonant circuit at that frequency. Now, if the wire length is doubled, the inductance and capacitance are *both* doubled and the L–C product is *four* times as great; if the wire length is halved, the inductance and capacitance are both halved and the L–C product is one-fourth as great. However, F varies *inversely as the square root of the L–C product.* Therefore, if the wire length is doubled, the frequency is halved; if the wire length is halved, the frequency is doubled.

c. EXAMPLES. When L is in μh (microhenrys) and C is in μf, the required L–C product for a frequency of 750 kc (kilocycles) is .0450 (from L–C table). Assuming that a .0005-μf capacitor is to be used,

$$L = LC/C$$

$$L = .0450/.005 = \frac{45 \times 10^{-3}}{5 \times 10^{-4}} = 90\ \mu\text{h}$$

If L and C are both doubled, *as is the case when a wire length is doubled,*

$$L = 2 \times 90 = 180\ \mu\text{h}$$
$$C = 2 \times .0005 = .001\ \mu\text{f}$$

and the new L–C product is

$$LC = 180 \times .001 = .180$$

From the L–C table, the L–C product .180 corresponds to a frequency of 375 kc. This is *one-half* the original frequency of 750 kc. If L and C are both halved, *as is the case when a wire length is halved,*

$$L = 90/2 = 45\ \mu\text{h}$$
$$C = .0005/2 = .00025\ \mu\text{f}$$

and the new L–C product is

$$LC = 45 \times 25 \times 10^{-5} = 1125 \times 10^{-5} = .01125$$

From the L–C table, the L–C product .01125 corresponds to a frequency of 1,500 kc. This is *twice* the original frequency of 750 kc.

d. VARIATION OF WAVELENGTH WITH WIRE LENGTH. As shown above, there is a definite relationship between wire length and frequency. Since a definite relationship also exists between frequency and the wavelength of an electromagnetic wave, there must be a relationship among the three factors, wavelength, frequency, and wire length. Consider the wavelength equation,

$$\text{wavelength} = \frac{\text{velocity}}{\text{frequency}}$$

where: wavelength (λ) is in meters
velocity, V, is the speed of light (approximately 300,000,000 meters/sec)
frequency F, is in cycles

When wavelength is given in meters and frequency is given in megacycles, this equation becomes

$$\lambda = 300/F_{mc}$$

Referring to the frequencies discussed in the examples above, when F is 750 kc (.75 mc),

$$\lambda = 300/.75 = 400 \text{ meters}$$

for F of 375 kc (.375 mc),

$$\lambda = 300/.375 = 800 \text{ meters}$$

and for F of 1,500 kc (1.5 mc),

$$\lambda = 300/1.5 = 200 \text{ meters}$$

e. NATURAL FREQUENCY. From the foregoing equations it might be thought that for self-oscillations at a frequency of 1,500 kc a wire would have to be 200 meters in length. Note, however, that to complete 1 *cycle* of oscillation each *point* of charge travels the wire length *twice* (fig. 37), once from its original position to the opposite end of the wire and once more in returning to its original position. Consequently, self-oscillations occur at a frequency which is such that the wire length is *one-half* the wavelength determined from the basic wavelength formula (*d* above). The frequency at which self-oscillations occur is called the natural frequency or the natural resonant frequency. The time required for 1 cycle of oscillation at the natural frequency is called the natural period of the wire.

f. CONVENIENT LENGTH EQUATIONS. It is sometimes convenient to express wavelength in feet. One meter equals 39.37 inches or 3.28 feet. Thus, when the frequency is in megacycles, the wave length (λ) in feet is

$$\lambda = \frac{3.28V}{F_{mc}}$$

but,
$$\lambda = \frac{3.28 \times 300}{F_{mc}}$$

therefore,
$$\lambda = \frac{984}{F_{mc}}$$

For the half-wavelength ($\lambda/2$) this equation becomes

$$\lambda/2 = \frac{492}{F_{mc}}$$

These equations give the length in feet when it is assumed that the wave or wire is in free space. In practical work a correction factor must be applied. This is discussed in Chapter 2.

g. ELECTRICAL AND PHYSICAL LENGTH. There are two ways of designating the length of a line:

(1) *Physical length.* This is the length of a line measured in length units such as inches, feet, yards, meters, miles, etc.

(2) *Electrical length.* This is the length of a line measured in terms of *wavelength* at the frequency normally used on the line.

h. LONG AND SHORT LINES. Power, telegraph, and telephone lines may be several hundred miles in length. Such lines are *long* in terms of distance, but (usually) are *short* in terms of wavelength. For example, 60 cycles is commonly used frequency for power transmission. At 60 cycles one wavelength (λ) in meters is

$$\lambda = \frac{3 \times 10^8}{F \text{ in cycles}} = \frac{3 \times 10^8}{60} = 5 \times 10^6 \text{ meters}$$

and the length in miles is

$$\lambda = \frac{\text{meters} \times (3.28 \text{ ft/meter})}{5280 \text{ ft}}$$

$$\lambda = \frac{5 \times 10^6 \times 3.28}{528 \times 10} = 3{,}100 \text{ miles}$$

Thus, a 100-mile line is only $\frac{100}{3100}$ or 1/31 wavelength at a frequency of 60 cycles. Such a line is *long* in terms of distance, but *short* in terms of wavelength. Now consider a line which is 328 feet in length and normally used in 30-megacycle circuits. In terms of distance, this line is extremely short when compared with the 100-mile power line discussed above. However, one wavelength at an operating frequency of 30 mc is

$$\lambda = \frac{300}{F_{mc}} = \frac{300}{30} = 10 \text{ meters}$$

The 328-foot line is $328/3.28 = 100$ meters in length and, therefore, its electrical length is $100/10 = 10$ wavelengths. In other words, a 328-foot line used at 30 megacycles is *electrically* longer than a 100-mile line used at 60 cycles.

30. Voltage and Current Distribution

Transmission lines cannot be considered as simple series circuits. In series circuits the current is everywhere the same as indicated by ammeters *A*, *B*, and *C* in figure 38. The current is *not* everywhere the same in the conductors of a transmission line. The constants *R*, *L*, *C*, and *G* are spread out or distributed along a wire (D, fig. 9) instead of concentrated as in simple series circuits (A, fig. 9). The *effects* of distributed constants differ from those obtained with lumped constants. Consequently, variations of current and voltage must be determined at each point along a transmission line.

a. INSTANTANEOUS VOLTAGE AND CURRENT. Although generally considered as an antenna, the resonant half-wave wire (pars. 28 and 29) provides an excellent example of the effects of distributed constants. Assume that oscillations are occurring in a wire length *A–B* (fig. 39). Also assume that the action can be stopped at times T_1 through T_{17}, so that the instantaneous

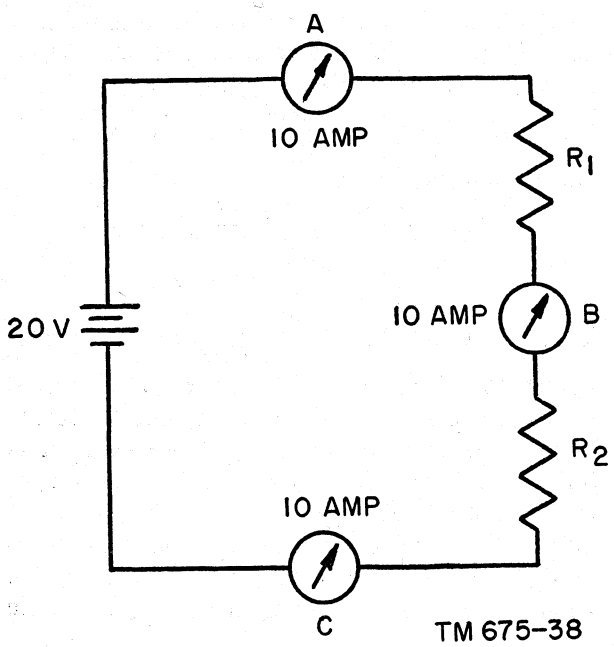

Figure 38. In a series circuit the current is everywhere the same.

voltages and currents can be measured at each point along the wire. Voltage and current distribution curves for each time T can then be made by plotting each reading against the point at which it is made on the wire.

(1) At time T_1 the points of maximum charge are at opposite ends of the wire, indicating that the distributed capacitance is fully charged. (This instant corresponds to time T_1 in figure 37.) The voltage *distribution* curve is sinusoidal with maximum potentials at opposite ends of the wire and zero potential at the center C. At this instant, there is no current at any point in the wire.

(2) The points of positive charge at A and of negative charge at B are *reflected* toward opposite ends of the wire (par. 25). At time T_2 each point of charge has moved 22.5° from its position at time T_1. (The arrows and degree markings below the wire indicate the direction and the distance (in degrees) that the corresponding point of charge has moved with reference to its position at time T_1.) The voltage distribution curve for this instant shows that the voltage is falling; the current distribution curve shows that the current is rising. Note that the current distribution is also sinusoidal and that the maximum current is at the center of the wire.

(3) Through times T_2 to T_5 the voltage decreases as the current increases. Zero voltage and maximum current occur at time T_5.

(4) After passing through zero at time T_5, the voltage reverses polarity and begins to rise, reaching its maximum value at time T_9. During this time the current decreases from maximum to zero but does *not* reverse. (Note that that this action through time T_9 corresponds to times T_1 through T_5 in figure 37.) The sinusoidal distribution shown in figure 39 is due to the distributed inductance and capacitance of the wire. Notice that neither the current nor the voltage is everywhere the same along the wire.

(5) At time T_9 the point of maximum negative charge is at A and the point of maximum positive charge is at B. *Reflection* again takes place and each point of charge moves back over the wire toward its original position at time T_1. Therefore, the voltage now decreases and the current reverses, rising from zero at time T_9 to maximum negative at time T_{13}. From time T_{13}, as the current decreases to zero at time T_{17}, the voltage reverses and rises to maximum at time T_{17}.

b. TRAVELING WAVES.

(1) *Individual waves.* At time T_1 (fig. 39) the point of maximum positive charge is at A and the point of maximum negative charge is at B. If the point of positive charge is considered *by itself*, it produces at this instant a certain distribution of charge and a corresponding voltage wave (dashed-line waveform in A, fig. 40). Similarly, considered by itself, the point of negative charge produces a different distri-

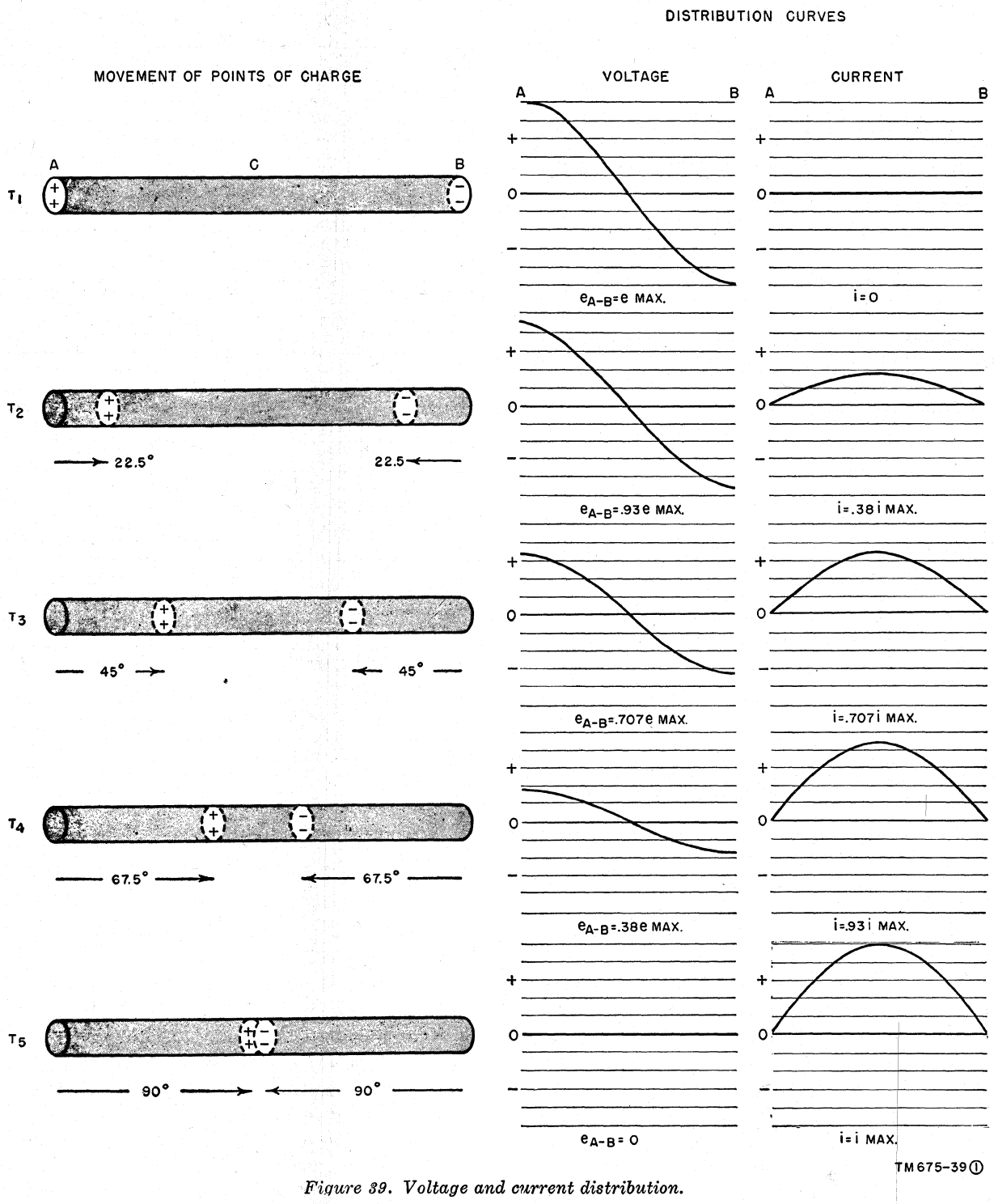

Figure 39. Voltage and current distribution.

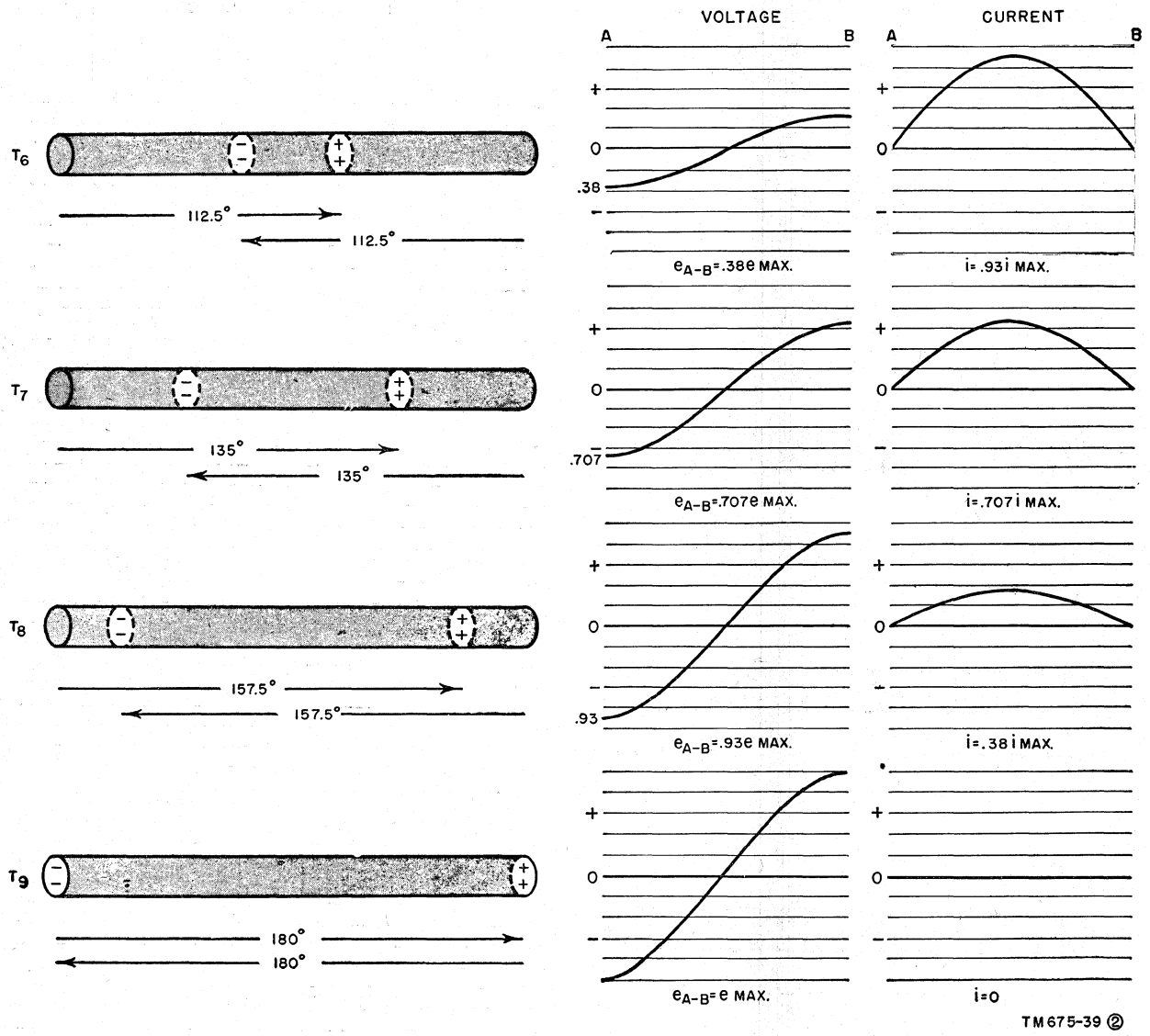

Figure 39—Continued.

bution of charge and, hence, a different voltage wave (dash-dot waveform in A, fig. 40). Since the points of charge travel along the wire, these two voltage waves also travel over the wire, and *at any instant the resultant voltage wave is the sum of these two traveling waves.* Assume that the concentration of positive charge at point *A* makes point *A* 10 volts positive with respect to the center of the wire. By itself the voltage wave due to the negative charge at point *B* also makes point *A* 10 volts *positive.* The resultant potential at *A* is the algebraic *sum* of these two potentials, or 20 volts. By similarly adding at each point the potentials due to each of the individual waves, the resultant voltage wave is determined (solid line waveform in A, fig. 40). At time T_3 in figure 39, the *points* of maximum charge have been reflected and have traveled 45° from the wire ends. The individual waves also have traveled 45° along the wire and are positioned as shown in B, figure 40. Addition of the individual waves at this instant pro-

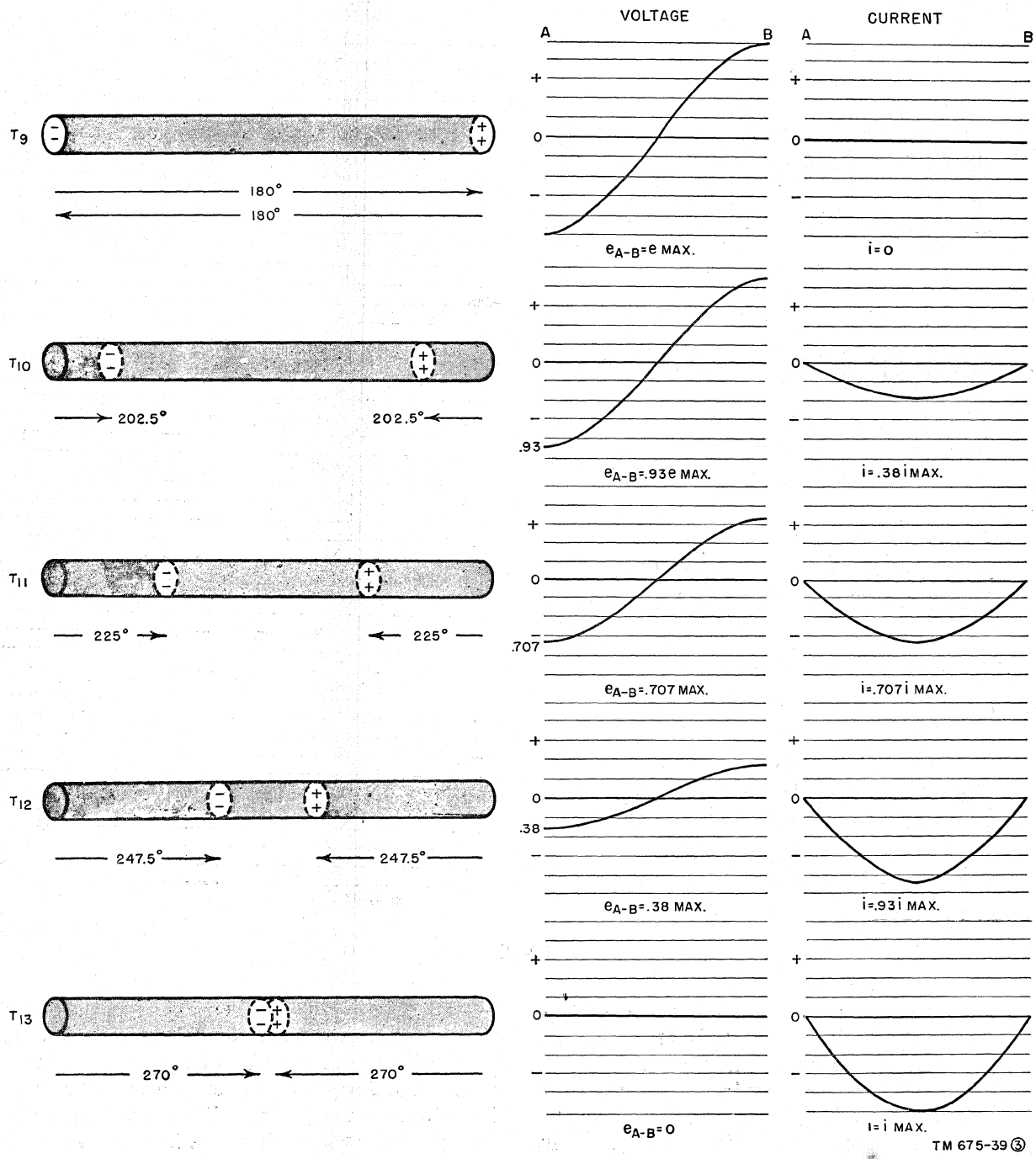

Figure 39—Continued.

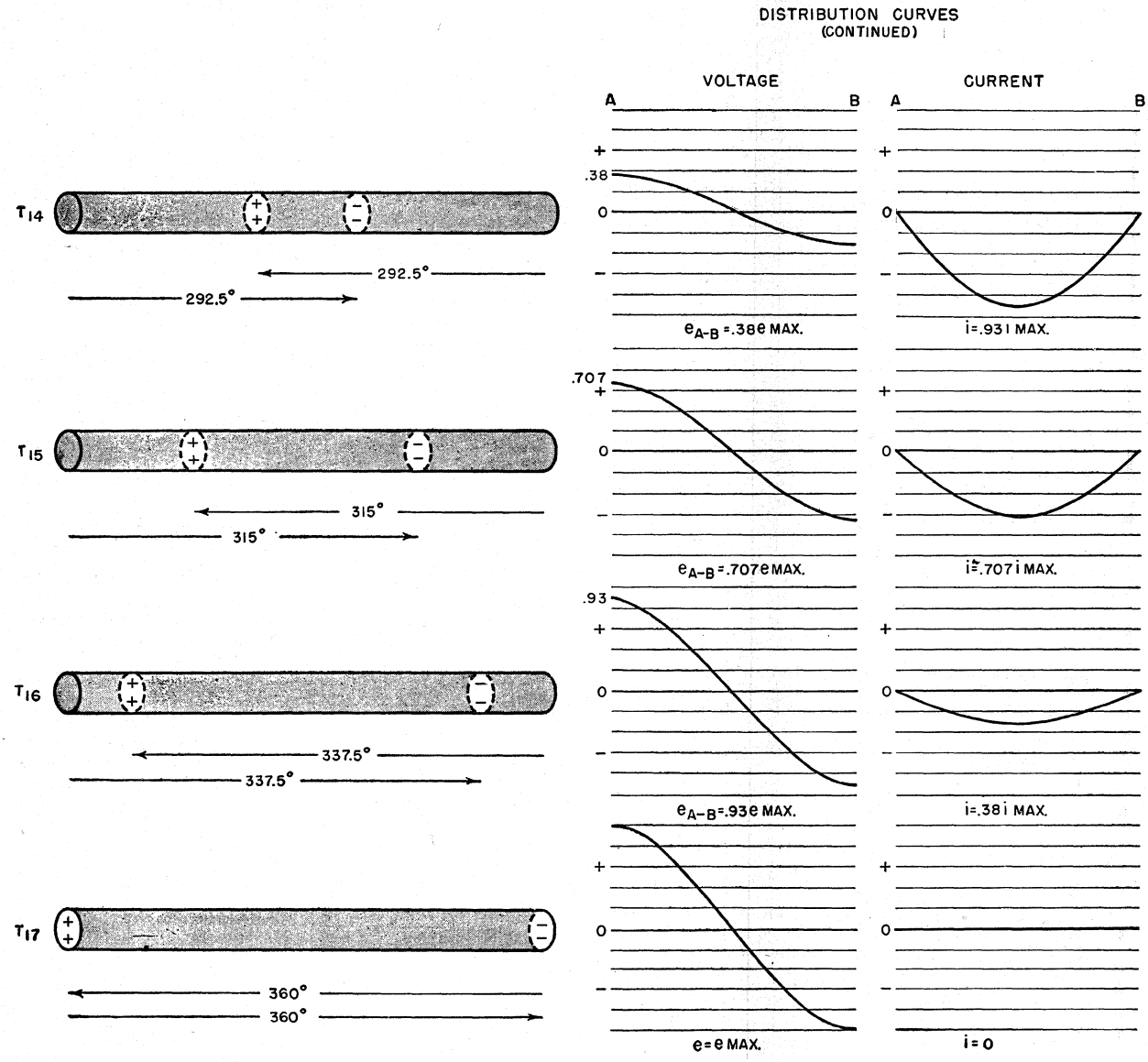

Figure 39—Continued.

duces a resultant voltage wave of 14 volts peak amplitude. When the points of maximum charge are passing one another (time T_5 in fig. 39) the individual voltage waves are exactly 180° out of phase (C, fig. 40). There is, therefore, no resultant voltage on the wire at this instant.

(2) *Traveling current waves.* Individual and resultant current waves for times T_1, T_3, and T_5 (fig. 39) are shown in D, E, and F, respectively, of figure 40.

c. SUMMARY. The current and voltage in a transmission line must be determined at each point along the line. The current is *not* everywhere the same as in a simple series circuit. Both the current and voltage are constantly changing at every point on a line. On any line in which *reflections* occur, the resultant instantaneous voltage, or current, at any point is the algebraic *sum* of two instantaneous voltages, or currents, at that point. These instantaneous voltages, or currents, are due to two individual waves which travel in opposite directions along

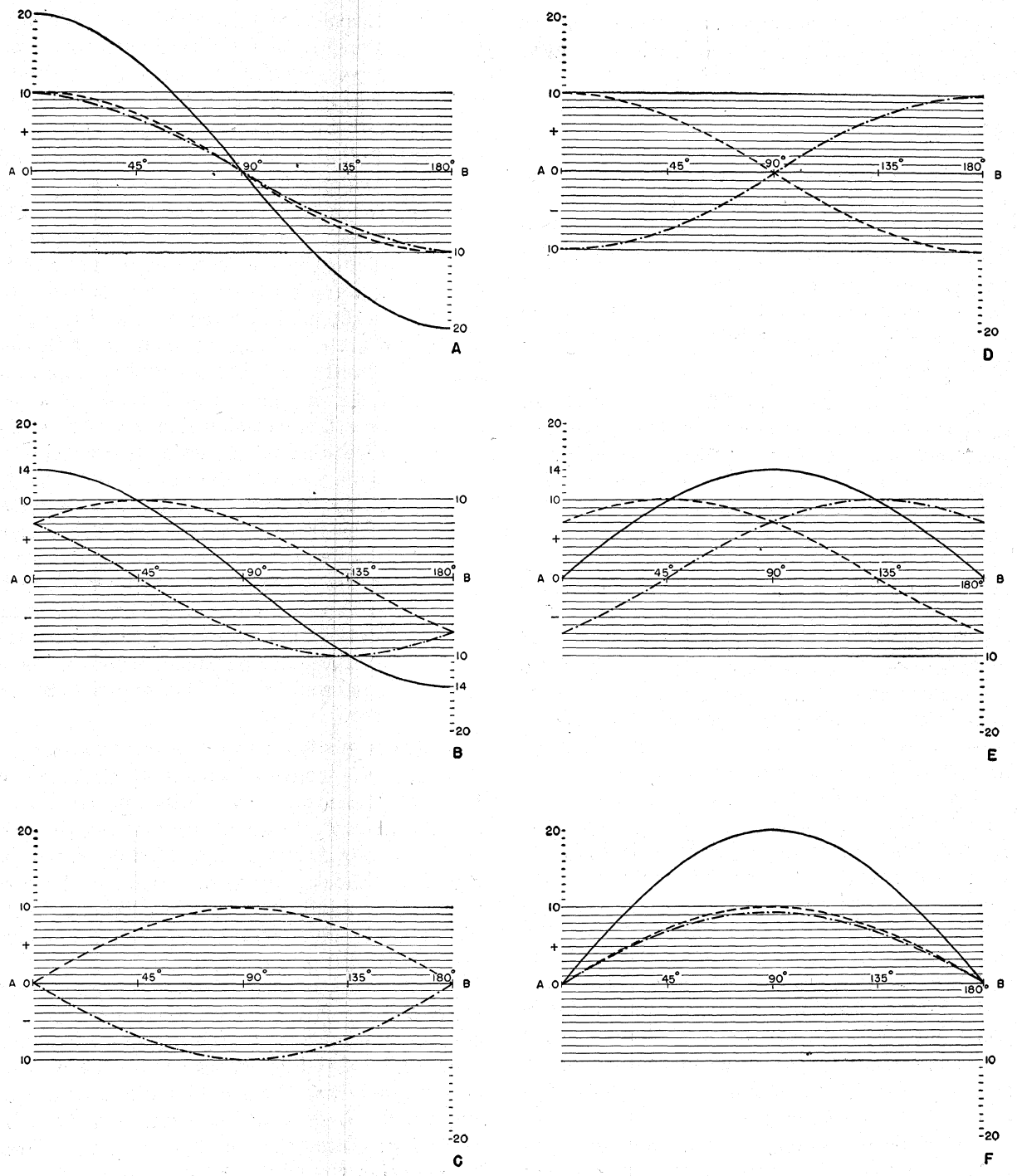

Figure 40. Reflected waves combine to produce resultant voltage and current.

the line. Voltage and current conditions in the absence of reflection are considered in detail in chapter 2.

31. Standing Waves

Probably the most important result of reflection is the production of *standing waves.* As discussed in paragraphs 25 and 30, reflection produces two elemental waves that travel in opposite directions over the wire. Because of this phenomenon, the resultant voltage and current distribution differs from that when there is no reflection.

a. VOLTAGE STANDING WAVE (VSW). In A, figure 41 a few of the instantaneous voltage distribution curves for ½ cycle of oscillation in a wire *A–B* are drawn on the same graph. This graph indicates how the voltage at each *point* varies. Thus, assuming a *peak* voltage of 10 volts and reading downward along the time axis, the voltage at point *A* is +10 at time T_1, +7.07 volts at time T_2, +3.8 volts at time T_3, 0 volts at time T_4, —3.8 volts at time T_5, —7.07 volts at time T_6, and —10 volts at time T_7. This completes the first half-cycle; the second half-cycle can be followed by reading upward from time T_7 to T_{13}. If a person now fixes himself in position to see *only* the voltage at point *A*, he will see a sinusoidal voltage (B, fig. 41). In other words if the voltage at point *A* is plotted against time on a separate graph, as in B, figure 41, it is found to be a sinusoidal voltage. Similarly plotted against time, the voltage at point *Y* also is found to be a sinusoidal voltage, but the peak amplitude is only 7.07 volts. By plotting the voltage at each point along the wire, it will be found that the voltage at every point is sinusoidal (except, of course, at the reference point *X*). However, the peak amplitude at any point is a function of the distance of that point from the reference point *X*.

(1) *Meter readings.* If an a-c meter is used to measure the voltage at each point along *A–B*, and if the meter readings are plotted against position on the wire, the solid-line voltage curve *E* of figure 43 will be obtained. An a-c meter does not show polarity; therefore, this curve is drawn without polarity indications. Also, because the measurements are usually made with a meter that reads in effective (root mean square) values, the amplitudes indicated in figure 43 are .707 of those indicated in B, figure 41.

(2) *Standing waves.* The solid-line curve in figure 43 represents a *standing wave* of voltage. In other words, *as measured with an a-c meter,* the voltage along the half-wave wire *seems* to be a wave that remains *motionless.* Actually, as indicated in A and B, figure 41, the voltage at each *point* is constantly varying. As will be explained later, the presence of standing waves on a transmission line is a positive indication that reflection is occurring at some point on the line (usually at the load). In transmission line applications standing waves are of considerable importance; they enable measurements of frequency and wavelength, are an indication of the amount of radiation that may be expected from the line, and are an excellent indication of how well the load is matched to the line.

b. STANDING WAVES OF CURRENT. Again referring to the length of wire *A–B*, a number of the instantaneous current distribution curves are shown in A, figure 42. By plotting the current at points *X* and *Y* against time, it will be found that the current at each point in the wire varies sinusoidally (B, fig. 42). Measurements with an a-c meter result in the dashed-line current standing wave shown in figure 43.

c. GENERAL. It should be borne in mind that the half-wave wire (figs. 41, 42, and 43) is used here as a means of explaining the cause and meaning of standing waves. It should also be remembered that these explanations of waves are *simplified.* In actual transmission lines the exact distribution of standing waves may differ considerably from that shown in these illustrations. Later discussions will show that the phase and amplitude relationships of voltage and current waves, as well as their positions on the line, are a function of the type of line, the type of load connected to the line, impedance matching, and other characteristics of the particular circuit arrangement.

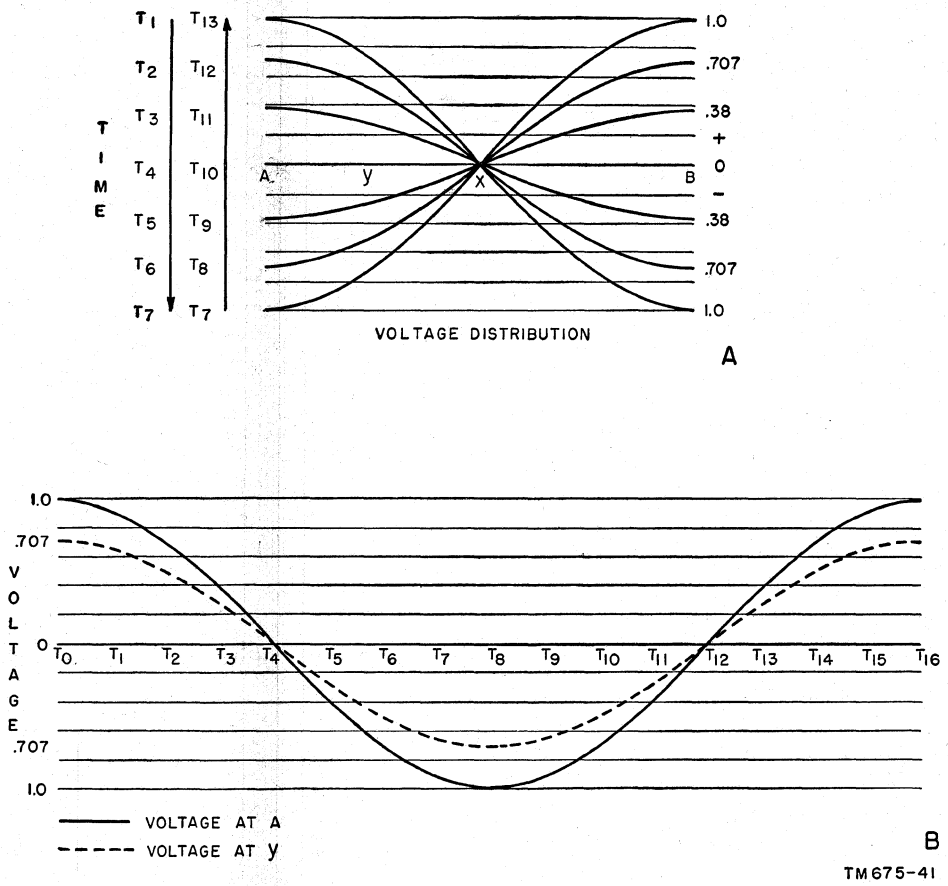

Figure 41. Voltage at each point is sinusoidal.

32. Transmission of Energy

In any electric circuit the ability to do work (energy) is in the electric field (par. 7). Basically then, *the problem in transmitting electric energy over appreciable distances is to bring the generated field to the load.* When the load is connected directly to the generator (fig. 25), the explanation that electrons (current) are forced through the wires by electrical pressure is reasonably satisfactory. This explanation is, of course, the familiar water analogy: A pump (electrical pressure) forces water (current) through pipes (wires). Such an analogy implies a continuous force through the wires and load, but there is actually no such continuous force over a transmission line. Consequently, transmission over distances must be explained in terms of electric waves or, more precisely, electromagnetic waves.

a. RIBBON WAVE. In figure 44, a battery is connected through a relatively long two-wire transmission line to a load at the far end of the line. At the moment that the switch is closed *neither voltage nor current is distributed over the line.* Point *A* becomes a point of positive potential and point *B* becomes a point of negative potential. As explained in section III, these points of potential move down the line. However, as the initial points of potential leave points *A* and *B*, they are followed by new points of potential which the battery adds to *A* and *B*. This is merely saying that the battery maintains a constant potential difference between *A* and *B*. A short time *after* the switch is closed, the initial points of potential have reached points A′ and B′ and the wire *sections A–A′* and *B–B′* are at the same potential as *A* and *B*, respectively. The points of charge are represented by + and — signs along the wires, the currents in

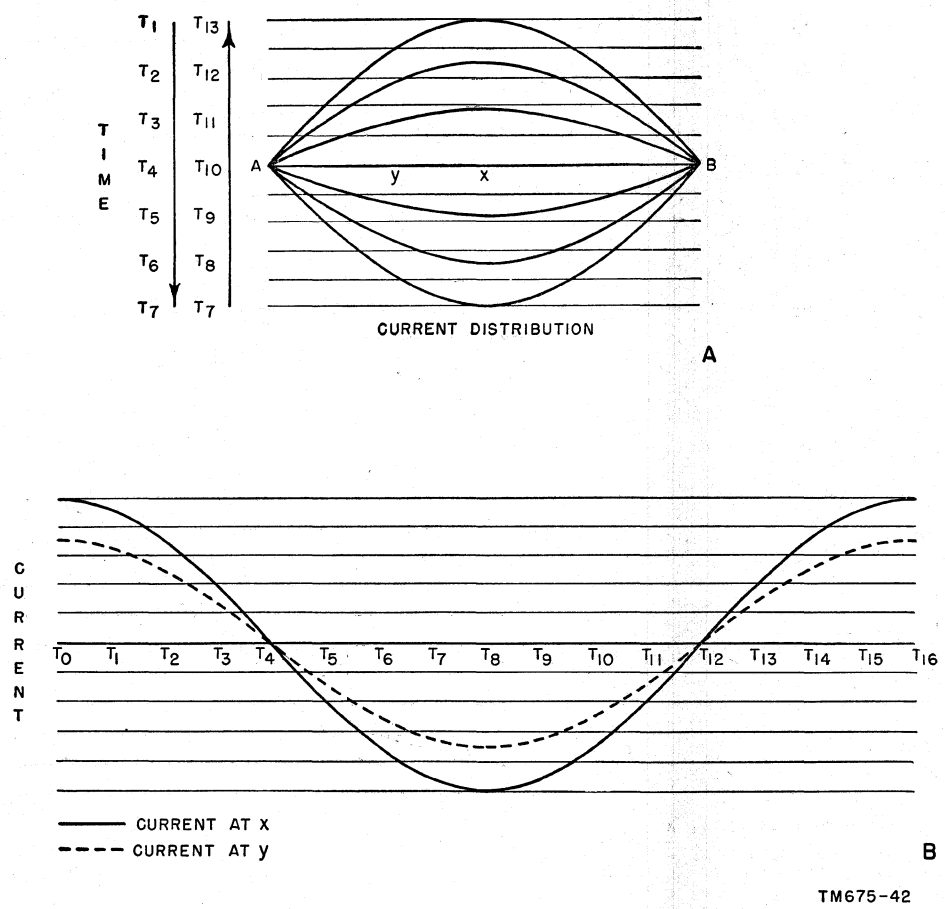

Figure 42. Current at each point is sinusoidal.

the wires are represented by short arrows, and the direction of travel is indicated by an arrow below the line. Conventional lines of force represent the electric field that necessarily exists between the opposite kinds of charge on *A–A′* and *B–B′*. Crosses (tails of arrows) indicate the magnetic field created by the electric field *moving* down the line. The moving electric field and its accompanying magnetic field constitute an electromagnetic wave that is moving from the generator (battery) toward the load. This type of electromagnetic wave is sometimes called a ribbon wave, because it moves down the line much as a ribbon would if inserted at the battery and pulled toward the load. This ribbon wave travels approximately at the speed of light in free space and, when it reaches the load, the energy reaching the load is equal to that developed at the battery. (This assumes no losses in the transmission line.) Assuming that the load absorbs all of the energy, there will be no reflection, and the current and voltage will be evenly distributed along the line.

b. SINUSOIDAL WAVES, INCIDENT WAVES. When the battery of figure 44 is replaced by an a-c generator (fig. 45), each successive instantaneous value of the generator voltage is propagated down the line at the velocity of light. The action is like that described for the ribbon wave, except that the applied voltage is sinusoidal instead of constant. Neglect possible transients and assume that the switch is closed at the moment that the generator voltage is passing through zero to make point *A* negative. At the end of 1 cycle of generator voltage the current and voltage distribution will be as shown in figure 45. In this illustration conventional lines represent the electric fields. For simplicity, the magnetic fields are not shown, but their directions are indicated by legends in the drawing. Points of charge are indicated by + and — signs, larger signs indicating points of higher ampli-

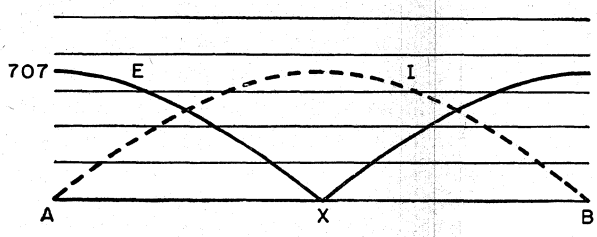

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Figure 43. Standing waves of voltage and current on half-wave wire.

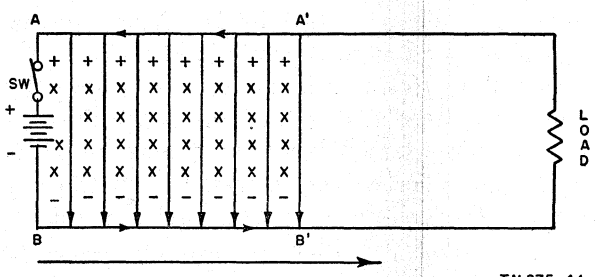

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Figure 44. D-c voltage applied to line.

tude of *both* voltage *and* current. Short arrows indicate directions of current (electron). The waveforms drawn below the line represent the voltage, E, and current, I, waves. It is assumed that the line is infinite in length so that there is no reflection. Thus, traveling sinusoidal voltage and current waves continually travel *in phase* from the generator toward the load or far end of the line. Waves traveling from generator to load are called *incident* waves.

33. Distribution of Energy

The discussion of oscillations in a half-wave wire (par. 28) showed that at certain instants all of the energy is in the electric field and that at certain other instants all of the energy is in the magnetic field. Examination of the fields between the conductors in figures 44 and 45 shows that part of the energy is in the electric field and the remaining energy is in the magnetic field.

a. Division of Energy. The maximum energy, W, stored in the inductance (magnetic field) per unit loop length is $W_L = LI^2/2$; the maximum energy stored in the capacitance (electric field) per unit length is $W_C = CE^2/2$. It can be shown that the impedance, Z_o, of a line is

$$Z_o = E/I = \sqrt{L/C}$$

from this, $E^2/I^2 = L/C$ (squaring)

and, $CE^2 = LI^2$ (cross multiplication)

but if $CE^2 = LI^2$, $W_L = LI^2/2 = CE^2/2 = W_C$

This means that the energy is equally divided

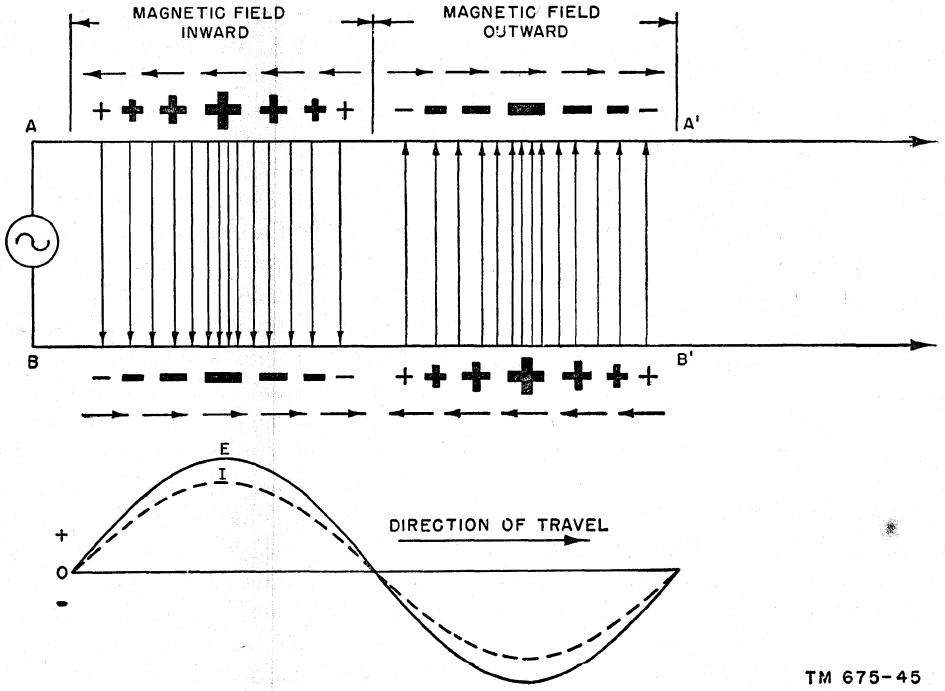

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Figure 45. A-c voltage applied to line.

between the electric and the magnetic fields. This assumes high Q lines; that is, WL/R is high.

b. VOLTAGE AND CURRENT. The foregoing equations indicate that the energy is in the electromagnetic field. This seems at variance with more familiar explanations in terms of voltage, current, and resistance. It might be well, therefore, to consider the relationship of fields and voltage and current.

(1) Electrons and protons are particles of electricity; they cannot be created or *generated* by man.

(2) Man can furnish energy to separate charges. The energy thus expended is then in the force (*electric field*) between the separated charges. Electric potential is a measure of the amount of work done in separating the charges, and the *field* will do the same amount of work in reestablishing an electrically neutral condition. Thus, voltage measurements in a transmission line are indirect measurements of the electric *fields* along the line.

(3) Energy in the electric fields about points of charge causes currents in the wire conductors. Thus, the potential differences and currents along a line are functions of the electromagnetic waves moving along the line.

(4) It is extremely difficult to make measurements in the electromagnetic waves themselves. However, line voltages and currents can usually be measured directly or calculated from measurements made with relatively simple equipment. Consequently, for most practical applications line theory can be studied in terms of voltages and currents.

34. Summary

a. ELECTROMAGNETIC WAVE. The action of any transmission is due to the basic phenomena that produce electric forces when electric charges are separated. If points of charge are established on a line, the resultant *field* causes current in the line and the points of charge move along the line. The moving field is an electric field in *motion* and produces a magnetic field; therefore, an electromagnetic wave travels along the line. Because most of the energy is in the electromagnetic wave that it guides from generator to load, a transmission line is actually a wave guide.

b. INCIDENT, REFLECTED, AND STANDING WAVES. When all of the energy in incident waves is absorbed by the load, no reflection can take place. However, if all of the incident energy is not absorbed by the load, the unabsorbed portion is reflected and travels back over the line toward the generator. Under this condition there are *two* waves on the line; incident waves travel from the generator to the load, and reflected waves travel from the load toward the generator. These incident and reflected waves combine to produce standing waves on the line.

c. FREQUENCY, WAVELENGTH, AND WIRE LENGTH. Owing to the distribution of L, C, and R along a line, there is a definite relationship among frequency, wavelength, and wire length. These relationships and the production of standing waves form a basis for many of the practical applications of transmission lines. Thus, lines are used as frequency and wavelength measuring devices, as one-to-one, step-up, step-down, and impedance matching transformers, as resonant circuits in oscillators and amplifiers, and as excellent insulators in radio-frequency circuits.

d. PRACTICAL EXPLANATIONS. Despite the fact that the *energy is in* the *electromagnetic waves* that the line is guiding, line operation is usually explained in terms of line voltages and currents. This is because it would be difficult to solve complex line problems by field theory, and the *voltage and current* equations are sufficiently accurate for engineering purposes. Throughout the remainder of this manual, therefore, lines will be discussed in terms of voltages and currents. The necessarily brief, simplified discussions in this chapter should provide a better understanding of the *why* and *how* of line action. A reasonably fair mental picture of the invisible action is always an aid to understanding the operation of any electrical device.

CHAPTER 2

PRACTICAL LINE THEORY

Section I. THE INFINITE LINE

35. General

Chapter 1 discusses transmission-line theory in terms of elemental charges, electric fields, magnetic fields, and electromagnetic waves. In this chapter the infinite line, characteristic impedance, lumped and distributed constants, and practical applications are considered in terms of line voltages and currents, and voltage and current waves.

36. The Infinite Line

The theoretical behavior of two parallel wire conductors of infinite length forms a starting point or basis for studying practical lines. Stated simply, this theoretical line is of importance for the following reasons: (1) Many lines, particularly power and telephone lines, are so long that they act approximately as infinite lines; (2) the transmission of electric waves over any line can be analyzed in terms of the infinite line; (3) any properly terminated line will act like an infinite line; and (4) the infinite line is the simplest to study.

37. Terminology

Every transmission line has two ends, the end to which power is applied and the end to which the load is connected to receive power. The first is called the sending end, the input end, or the generator end; the second is called the receiving end, the output end, or the load end. The opposition of a line to current from a generator is called the input impedance, symbol Z_{in}. Input impedance is expressed by the equation $Z_{in} = E/I$, where E is the voltage and I is the current at the input terminals of the line. The symbol Z_r is used to indicate the impedance of the load at the receiving or load terminals of the line. Other terms and symbols will be introduced as required in the text.

38. Characteristic Impedance

The input impedance of any real line is a function of frequency and the load impedance. For any line there is one, *and only one,* value of load impedance that will make the input impedance equal to the load impedance. The required value of load impedance is equal to *the input impedance of an infinite length of the particular line.* This value of impedance is called the *characteristic impedance.* The concept of characteristic impedance is of particular importance. When terminated at its load end by a value of Z_r equal to its characteristic impedance, symbol Z_o, any line has an input impedance equal to its characteristic impedance; that is, $Z_{in} = Z_r = Z_o$. When $Z_{in} = Z_r = Z_o$, the impedance (looking toward the load end) at *every point* in a line is also equal to Z_o, the load impedance absorbs all of the energy reaching it from the generator, there are no reflections and, hence, no standing waves on the line. These ideas of *impedance matching* are of considerable importance and will be considered throughout this manual.

39. Fundamental Concept

According to the concept of infinity, an infinite line is so long that energy applied at the near end will never reach the load or far end of the line. If none of the input energy ever reaches

the load end of a line, it is feasible to assume that, regardless of its value, the load impedance Z_r will have no effect whatsoever on the input impedance Z_{in}. Suppose, for example, that a length of 500 miles is cut from either end of an infinite line. Whether the length is cut from the near or from the far end of the line, the generator *sees* no difference in the line, because the line is still infinite in length. Since neither the load impedance nor the cutting away of a length has any effect, we deduce that *the input or characteristic impedance of an infinite line is a function of line construction and frequency only.* Carrying this deduction a little further, we conclude that *the characteristic impedance of any line is determined entirely by frequency and the line construction:* the size, shape, spacing, number, and electrical characteristics of the conductors, and the electrical properties of the dielectric material.

40. D-c Voltage Applied to Infinite Line

In paragraph 32*a* the development of a ribbon wave is discussed in terms of the electric field and its effects on charges in the wires. We will now consider this same wave in terms of voltage and current.

a. PROPAGATION. The distributed inductance and capacitance of a line are represented in A, figure 46, by a series of extremely small lumps, the line resistance R and the shunt conductance G are neglected, and the line is assumed to be infinite in length.

(1) *Voltage wave.* At the moment that the switch is closed, the battery voltage E is applied to the input terminals of the line. At this same instant capacitor C_1 contains no charge and, therefore, it is a *short circuit* across *A–B*. Consequently, current I begins to flow through L_1 into C_1, *but not beyond A–B*. As current I flows into it, C_1 becomes charged and a voltage appears across *A–B*. Note that in this lumpy circuit, L_1, *delays* the charging of C_1 and no current can flow beyond *A–B* until C_1 acquires *some* charge. As C_1 charges, the voltage across it increases and causes current through L_2 into C_2. As C_2 charges, its voltage increases and causes current through L_3 into C_3, and this action continues down the line. First C_1 charges to voltage E, then C_2, then C_3, and so on. In other words there is a voltage wave traveling along the line. This wave starts at the sending end and travels with a certain velocity V. All of the line that the wave has reached is charged to voltage E (B, fig. 46). *Beyond the wavefront the line is uncharged.*

(2) *Current wave.* The voltage wave is necessarily accompanied by a current wave, I (B, fig. 46). To charge C_1 a current I must flow through L_1, to charge C_2 a current I must flow through L_2, and so on.

(3) *Voltage and current waves sustain one another.* For the voltage wave to travel down the line, current I must flow through each L into the following C, but for a current I through the next L a voltage must first be built up across each C. This means simply that the voltage and current waves are mutually sustaining. This idea is in agreement with the theory (TM 11–681) that a moving electric field creates a magnetic field and a moving magnetic field creates an electric field. The concept also agrees with the earlier statement (par. 17) that a magnetic field is an attribute or a different aspect of an electric field.

b. GENERAL. The previous discussion has been simplified by neglecting the line resistance and shunt conductance. Certain effects that occur in lumped circuits also have been neglected. For example, the voltage across C in a lumped circuit does not build up to E and remain constant; damped oscillations occur and the voltage across C is alternately greater and smaller than the battery voltage E. However, these effects become negligible as real line conditions are approached by reducing the size and increasing the number of lumps.

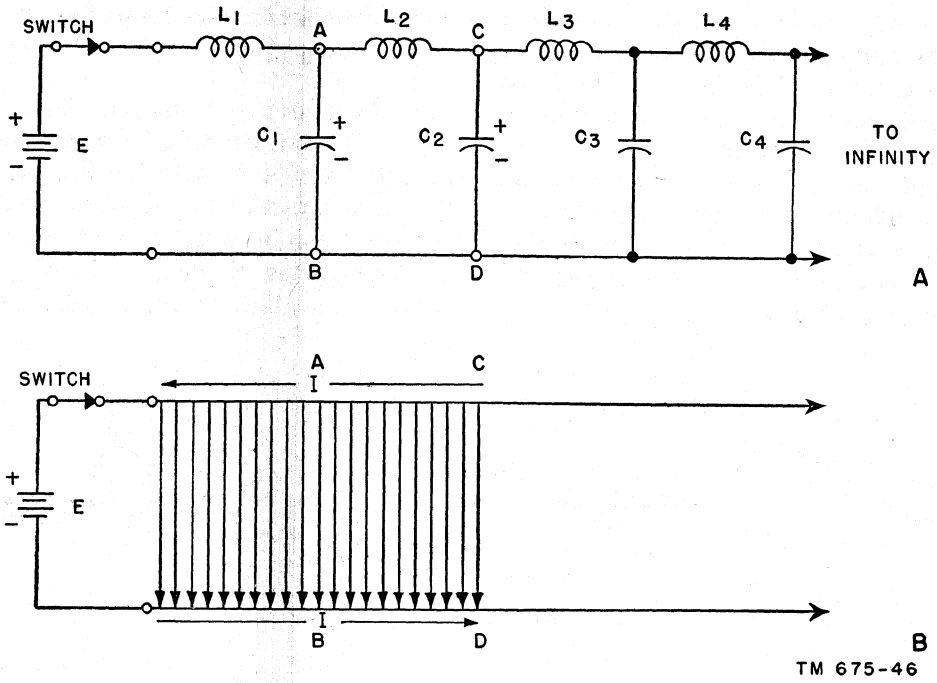


Figure 46. Voltage and current waves sustain one another.

Section II. PRACTICAL CONSIDERATIONS

41. Lumped and Distributed Constants

a. When a transmission line is short as compared with the length of the radio waves which it carries, most of the energy supplied to the input terminals is absorbed by the load, a small amount being lost in the resistance of the line. When the line is long as compared with one wavelength and if the load is not correctly chosen, the voltage, E, required to drive a given amount of current or power over the line may differ greatly from that calculated by using the impedance of the load, R_1, in series with the resistance of the line, R_2; that is, the voltage, E, does not equal $I\ (R_1 + R_2)$. In addition to its resistance the line has other properties which affect the input impedance. These properties are inductance in series with the line, capacitance across the line, conductance paths across the line, and certain radiation losses.

b. In ordinary circuits using coils and capacitors, inductance and capacitance are present in definite lumps. In r-f transmission lines, however, these quantities are distributed throughout the entire line and cannot be separated from each other.

c. A, figure 47, shows that any length of wire has electrical resistance. This resistance may be expressed in ohms per foot, in ohms per mile, or in other ways, depending on the size of the circuit under consideration.

d. B, figure 47, shows that any wire has inductance. A current in the wire sets up magnetic lines of force which encircle the wire. This magnetic field contains energy which, when the field collapses, is returned to the circuit. This property of inductance along a length of wire can be expressed in microhenrys per foot, in millihenrys per mile, or in some other convenient way.

e. C, figure 47, shows that an appreciable amount of capacitance exists between wires which are relatively close together. This is because the wires act as the plates of a capacitor, with the air or other insulating material between them acting as the dielectric. The capacitance of a line usually is expressed in micromicrofarads per foot or in microfarads per mile.

f. D, figure 47, shows that two wires close together have current leakage paths between them. These paths exist because no insulator is perfect and, therefore, there is a certain amount of conductance from one wire to the other. The property of conductance, symbol G, is usually expressed as the reciprocal of resistance, or $G = 1/R$. The conductance of a line usually is given in micromicromhos per foot or in micromhos per mile.

42. Characteristic Impedance and Infinite Line

a. Many unit lengths may be considered to form a longer length of transmission line (A and B, fig. 49). In most r-f lines the effects of the conductance G are very small compared with those of the inductance L and the capacitance C. The resistance R of a well-designed r-f line usually is negligible. In C, figure 49, G is omitted and L and R are treated as if they were concentrated in one side of the two-wire line. This simplification does not affect the solution of r-f line problems. The circuits shown in figure 49 are good approximations of real transmission line conditions, provided the sections are very small and sufficiently numerous.

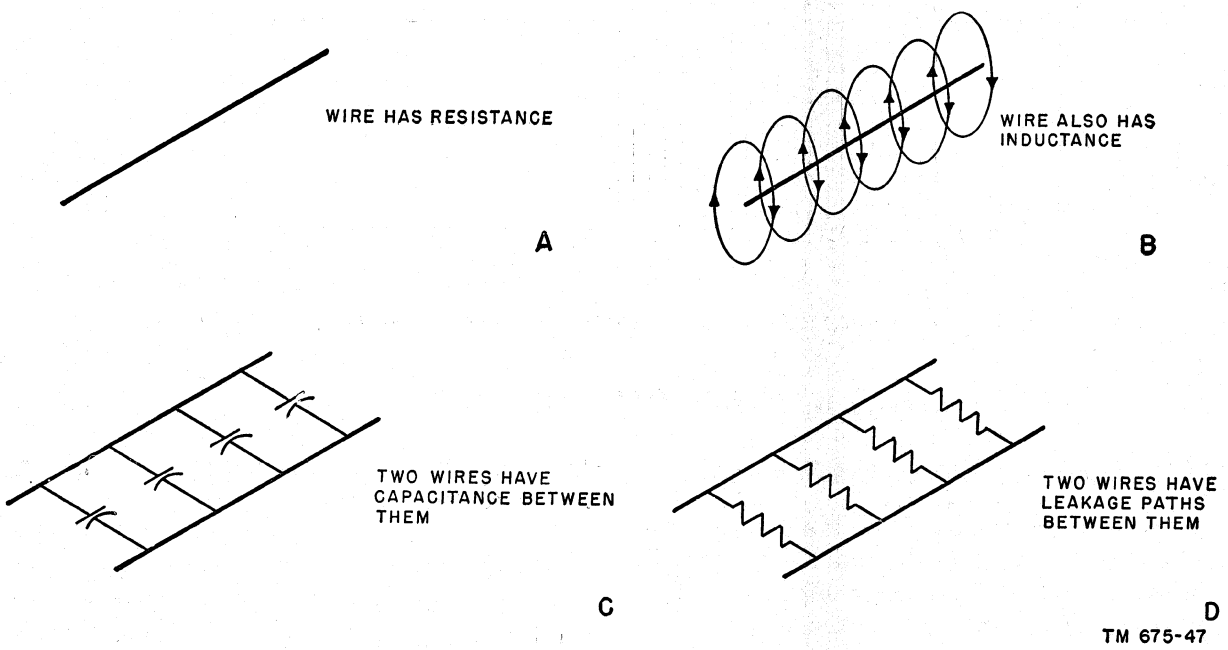


Figure 47. Electrical properties of wire.

g. A, figure 48, shows these properties of resistance R, inductance L, capacitance C, and conductance G combined in a short section or *unit length* of two-wire transmission line. Actually, the diagram does not represent a real transmission line, because it shows evenly *distributed* capacitance as a single lumped capacitance C, evenly *distributed* conductance as a lumped leakage path G, evenly *distributed* series inductance L as a lumped inductance, and evenly *distributed* series resistance as a lumped resistance. However, if the section is very short compared with one wavelength of the line, this approximation is good enough for practical purposes. B, figure 48, shows all four properties represented by conventional symbols.

b. Current flows into the line, if voltage is applied across the input terminals A and B in figure 49. The ratio of the voltage to the current is the input impedance, Z_{ab}, of the line; that is, $Z_{ab} = E/I$. The input impedance at the input terminals of a transmission line consists of more than the series resistance of the wires in series with the impedance of the load. The effects of the series inductance, shunt capaci-

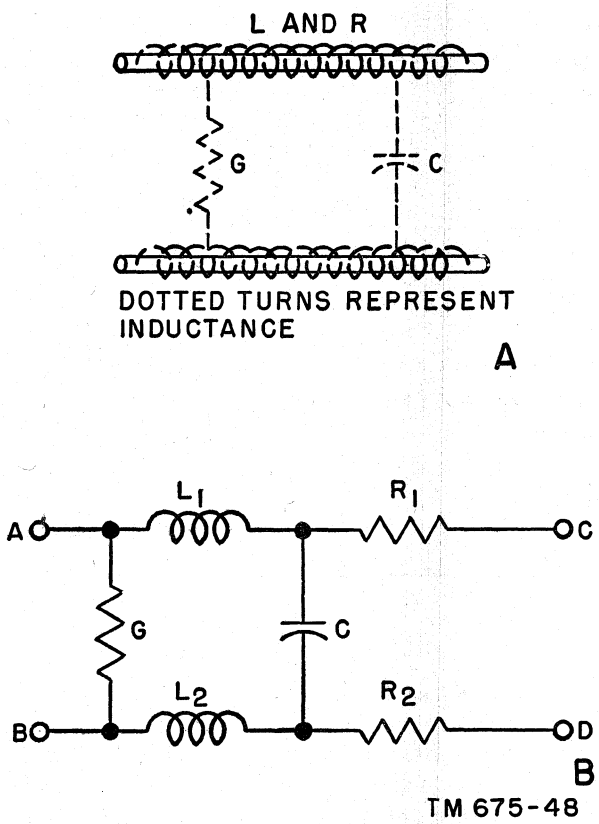


Figure 48. Representation of electrical properties in unit length.

tance, and shunt conductance may be considerably greater than the effects of the series resistance of the wires and the load impedance.

c. Assuming that the sections of C, figure 49, continue to the right indefinitely, the line is an infinite line and its input impedance Z_{ab} is equal to the characteristic impedance Z_o; that is, $Z_{ab} = Z_o$. If the near end of this infinite line is cut off at *MN* (A, fig. 50), the line is still an infinite line, because an infinite number of sections remain to the right of *MN*. This indicates that cutting a section from the near end of an infinite line has no effect on the input impedance; therefore, in A, figure 50, the input impedance Z_{MN} is equal to the characteristic impedance Z_o. If the first three sections are terminated by a load impedance equal to Z_o, the input impedance of the three sections is also equal to Z_o (B, fig. 50).

d. The concept of characteristic impedance, Z_o, at the input terminals of an infinite line has a very practical use. If *any* line is terminated by a load impedance equal to its characteristic impedance, the input impedance is equal to the characteristic impedance.

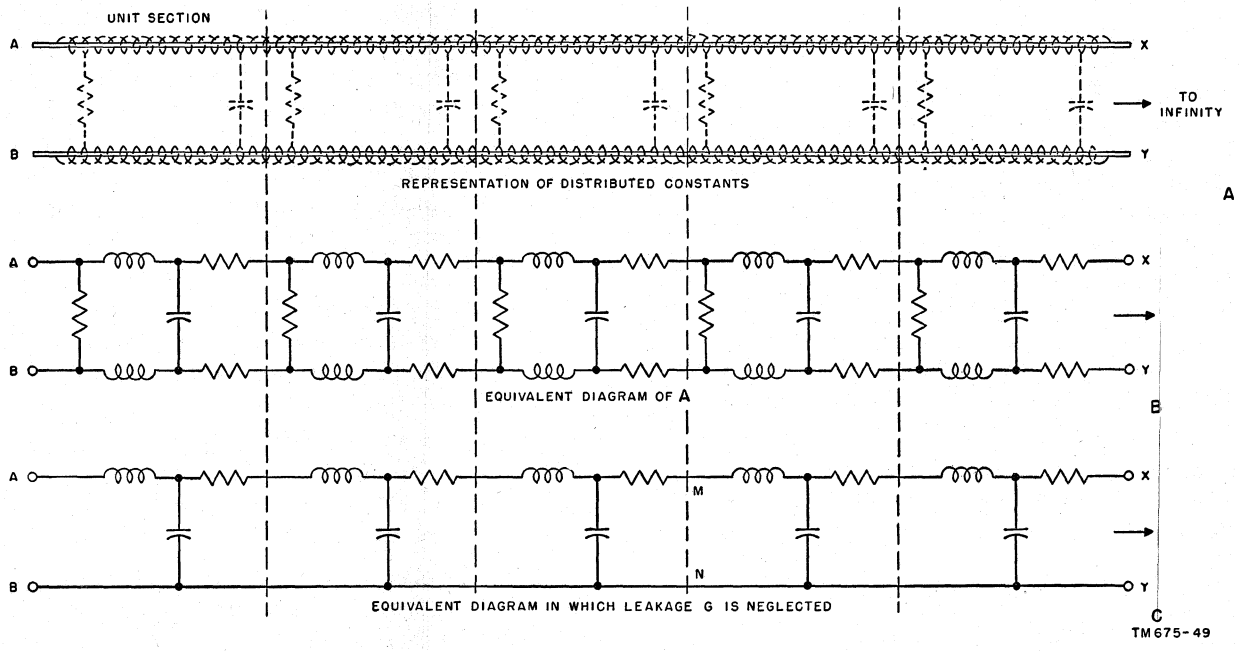


Figure 49. Representation of infinite line as unit lengths in tandem.

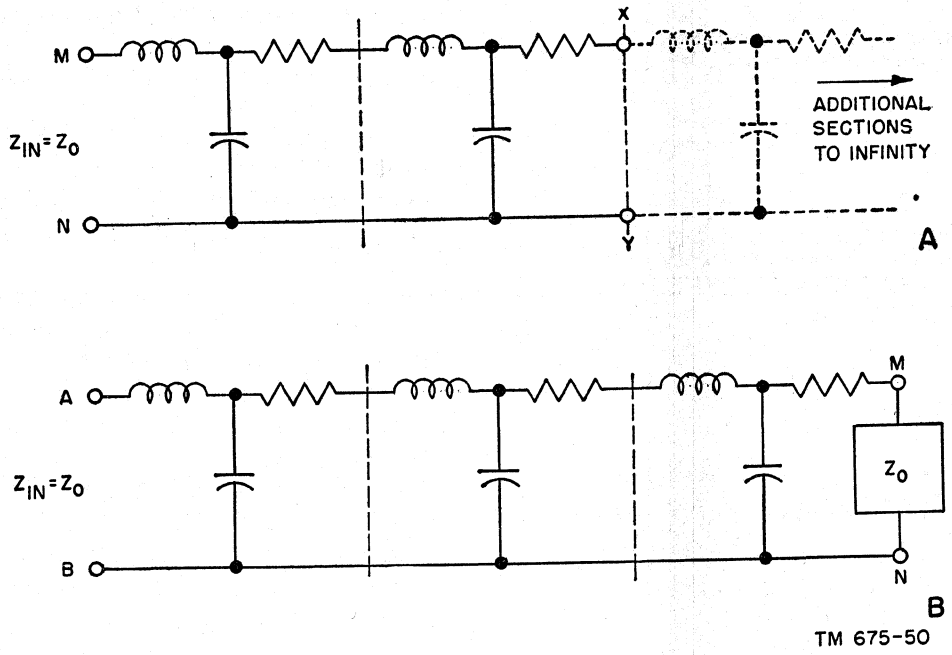


Figure 50. Z_{in} equals Z_o if line is infinite, in A, or terminated by Z_o, as in B.

43. Numerical Example

a. Figure 51 gives a numerical example of the way in which small identical sections of line can be added in tandem to make the input impedance approach the characteristic impedance. In A any voltage (direct or alternating) causes a certain current into the network, and the ratio $E/I = Z_{AB}$, the input impedance. Assume that a d-c (direct current) voltage of 100 volts is applied to the input terminals AB. In B, figure 51, a section consisting of a 10-ohm resistor in series with a 100-ohm resistor has an effective resistance of $10 + 100 = 110$ ohms, and the current, $I = E/R$, is .91 ampere. In C an identical section is added, so that the circuit consists of a series 10-ohm resistor followed by 110 ohms in parallel with the first 100-ohm resistor. By the rules for calculating series-parallel resistances, the input impedance, $Z_{AB} = E/I$, is now 62.4 ohms. Because of the lower input impedance, the current from the generator increases from .91 ampere to 1.6 amperes. In D a third section is added and the input impedance falls to 48.4 ohms.

b. Notice that each time a new section is added (E, F, and G, fig. 51) the input impedance Z_{ab} is reduced and the current is increased. However, the *change* in impedance and current becomes smaller and smaller as each section is added. In H, figure 51, the input impedance is only 37 ohms and additional sections may now be added without causing *appreciable* changes in the value of Z_{ab} and I. Therefore, the 37-ohm input impedance may be considered as the characteristic impedance of the network.

c. If the first section (B, fig. 51) is terminated by a resistance of 37 ohms (A, fig. 52) the input impedance, Z_{AB}, will also be 37 ohms and the current will be the same (2.7 amperes) as in H, figure 51. In fact, if any number of sections is terminated by a 37-ohm resistance (B, fig. 52), the input impedance will be 37 ohms. From these discussions we arrive at the following conclusions:

(1) *Any very long line assumes a definite input impedance approximately equal to the characteristic impedance of the line.*

(2) *If any line is terminated by a resistance equal to its characteristic impedance, the input impedance is equal to the characteristic impedance.*

d. For simplicity of explanation, a d-c voltage is applied and resistors are used as impedances in the previous example. In any real line there is, of course, inductance, capacitance, and shunt

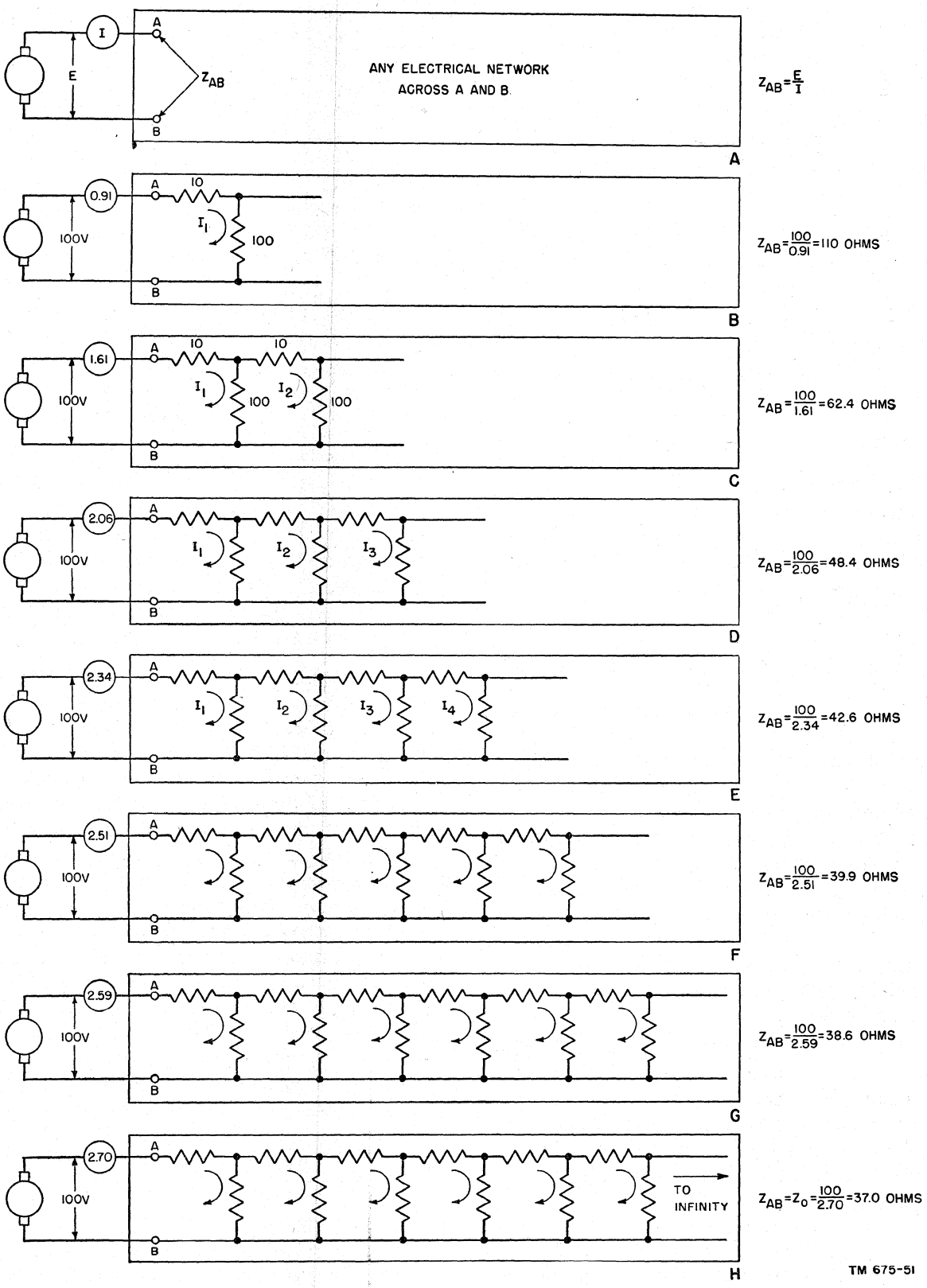


Figure 51. Z_{in} approaches Z_o as unit lengths are added to network.

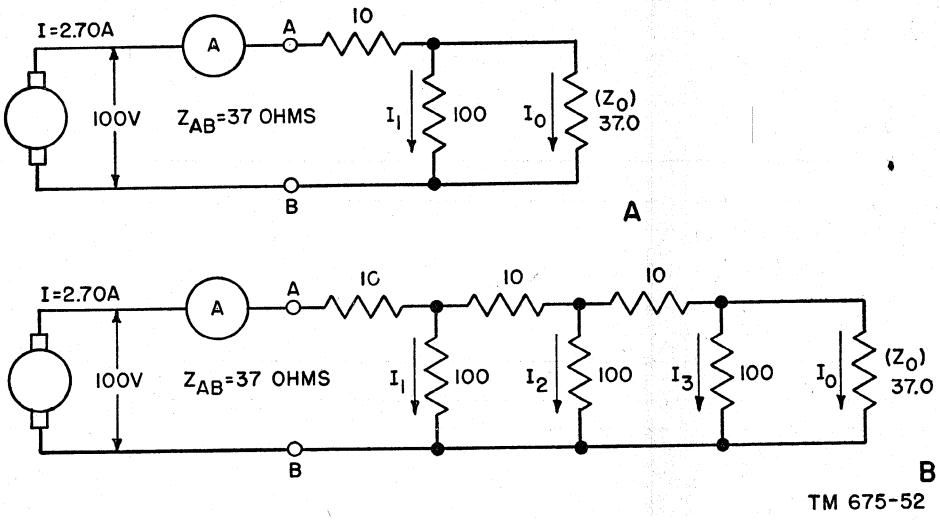


Figure 52. Termination of any line in Z_o.

conductance, as well as series resistance. However, it can be shown that the behavior of a real line, with a-c (alternating-current) voltages and currents, is like that of the network described here (fig. 51).

44. Wave Motion on an Infinite Line

a. WAVES. Figure 53 shows sine waves of voltage and current on an infinite line. The points of voltage maximum and current maximum are together, indicating that the voltage and current are in phase. Because of line losses, the voltage and current decrease in *amplitude* as the waves progress down the line. This figure illustrates what an observer would see if the voltage and current could be photographed at a particular instant of time, comparable to stopping a movie film at a single picture. An instant later the waves would have moved to the right slightly.

b. WAVES AND TIME. If a pebble is dropped into a pool of water the disturbance does not reach the edge of the pool immediately. Rather, a wave of water starts out from the place where the pebble hits and proceeds toward the edge at a definite speed. In electrical circuits as used in commercial power work, the time which electrical disturbances take to travel to another point on a wire is usually so small that it can be neglected. For example, the fraction of a micro-second which elapses between the time a lamp switch is closed and the time the electrons in the bulb are affected by its closing usually is ignored. In transmission line applications, however, it may not be possible to ignore this time, particularly when wavelengths are short compared with the line length.

c. VELOCITY OF ELECTROMAGNETIC WAVES. Electromagnetic waves in free space travel at the speed of light in free space (186,000 miles per second, or 300,000,000 meters per second). In an open two-wire transmission line the velocity of an electromagnetic wave is nearly equal to its velocity in free space. From this we see that, if a 30-megacycle voltage is applied to the input of a two-wire line, the first positive voltage and current peaks travel only 10 meters (32.8 feet) along the line before the next positive voltage peak is applied to the line. This means that at 30 megacycles one wavelength is approximately 32.8 feet long (B, fig. 54); at 3,000 megacycles one wavelength is only 4 inches in length (A, fig. 54); at 186,000 cycles one wavelength is almost a mile in length (C, fig. 54); at 1,000 cycles one wavelength is about 186 miles in length (D, fig. 54); and at 60 cycles (power line frequency) one wavelength is 3,100 miles in length (E, fig. 54).

d. DISTRIBUTION OF VOLTAGE AND CURRENT. In a wire carrying a 30-megacycle current (B, fig. 54) the currents in successive half-wave segments of the wire are in opposite directions.

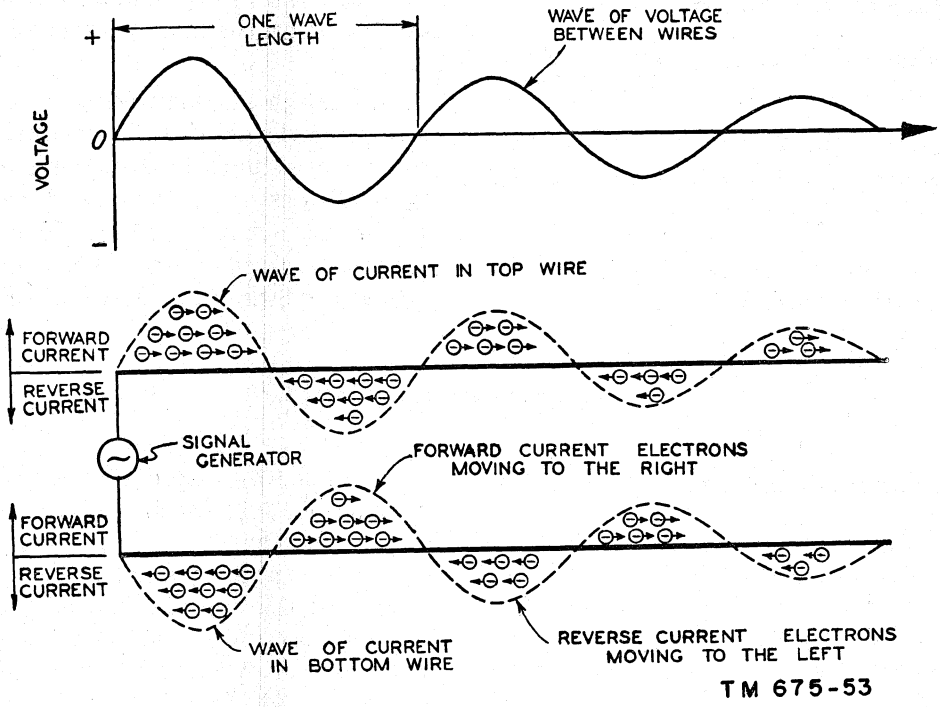


Figure 53. Voltage and current distribution on infinite line.

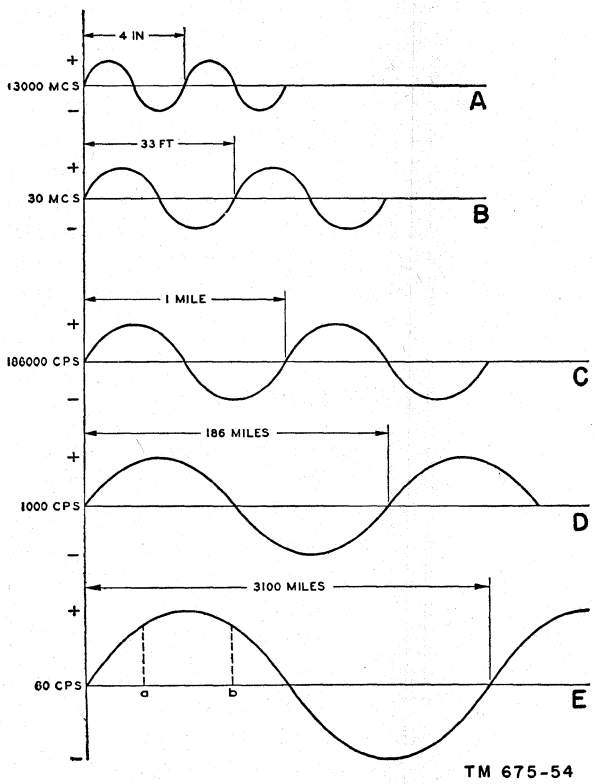


Figure 54. Voltage (or current) waves at various frequencies.

A small fraction of a second later the entire wave is a little farther along the line, because the waves are traveling continuously down the line, like the waves of water in a pool. E, fig. 54, shows that at a frequency of 60 cycles one wavelength is equal to 3,100 miles and the voltage (and current) is in the same direction along a 1,550-mile interval. For example, the distance from *a* to *b* in E, figure 54, might represent a 500-mile power line. Such a small part of the wave can be on this 500-mile length at any given instant that the voltage and current at the output are very nearly in phase with the voltage and current at the input. Reversals on the stopped wave should not be confused with the voltage and current reversals which take place at any *point* on the wire as a complete wave goes by. In the first case the observer stops the clock and examines the entire wave of current up and down the wire. In the second case he observes the voltage and current variations at *one point on the wire* as a wave passes that point. In an infinitely long line (fig. 53) or in any line terminated by a resistance equal to the characteristic impedance, the voltage and current are in phase at every point in the line.

45. Summary of Infinite Line

a. The voltage and current are in phase throughout the line.

b. The ratio of the voltage to the current, E/I, is constant over the entire line and is known as the characteristic impedance, Z_o.

c. The input impedance, Z_{in}, is equal to this characteristic impedance, Z_o.

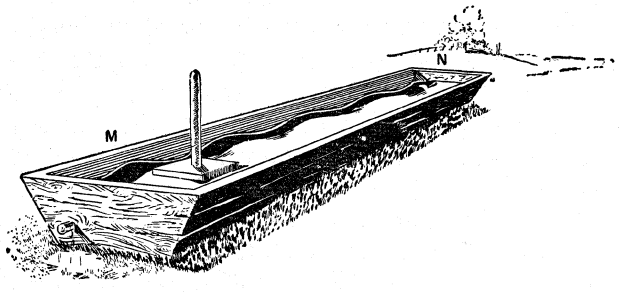
d. Since the voltage and current are in phase at every point, the line operates with minimum loss.

e. Although an infinite line is impossible to attain, knowledge of its theoretical behavior is of great assistance in determining the operating conditions of actual lines. A very long line behaves like an infinite line. When terminated by a resistance equal to its characteristic impedance, any length of line acts like an infinite line.

Section III. REFLECTION

46. Reflection on Open Line

a. CAUSE OF REFLECTION. The impedance at the output end of an open-circuited or open-end line can be considered as infinite. When energy is first applied to the generator end of the line, a wave of current and a wave of voltage sweep down the line in phase. These initial current and voltage waves remain in phase until they reach the infinite impedance at the open end of the line. On reaching this open circuit, the current must collapse to zero, because electrons cannot move beyond this point. When the current wave collapses, its magnetic field must also collapse. The collapsing magnetic field cuts the conductors near the end of the line and induces a voltage across the line. This induced voltage acts, in a way, like a reverse generator and sets up new current and voltage waves which travel back over the line toward the generator. Waves traveling from the generator toward the load are called *incident* waves; waves traveling back over the line toward the generator are called *reflected* waves.



TM 675-55

Figure 55. Reflection of water waves.

b. WATER ANALOGY. Reflection on a line may be compared with the reflection of water waves in a trough (fig. 55). When energy is applied to the water at M by a flat paddle, a wave travels toward the opposite end of the trough. If the trough extended to infinity, the waves would travel forever, without reflection. However, the trough ends at N and a wave of water splashes up on the end board, raising a certain amount of water *above* the average level. As this water falls, it delivers energy to a new water wave which travels back toward M. The electric wave and the water wave actions both produce *reflected* waves. Since the current is zero and the voltage is high at the output end of an open-circuited line, the resultant voltage and current waves are 90° out of phase as soon as the initial waves are reflected. Figure 56 shows this 90° phase difference between the voltage and current after reflections are set up in an open-circuited line.

47. Standing Waves of Voltage

Incident and reflected waves produce standing waves (fig. 57). Incident voltage and current waves travel down the line with a certain velocity and the reflected waves travel back at the same velocity. Incident voltage waves are reflected without a reversal of polarity. At each point on the line the incident and reflected voltage waves combine, or add, to produce a resultant voltage which is the sum of the two. The exact manner in which the two waves combine at any point is a function of the distance of that point from the point at which reflection starts. If the resultant voltages are measured and plot-

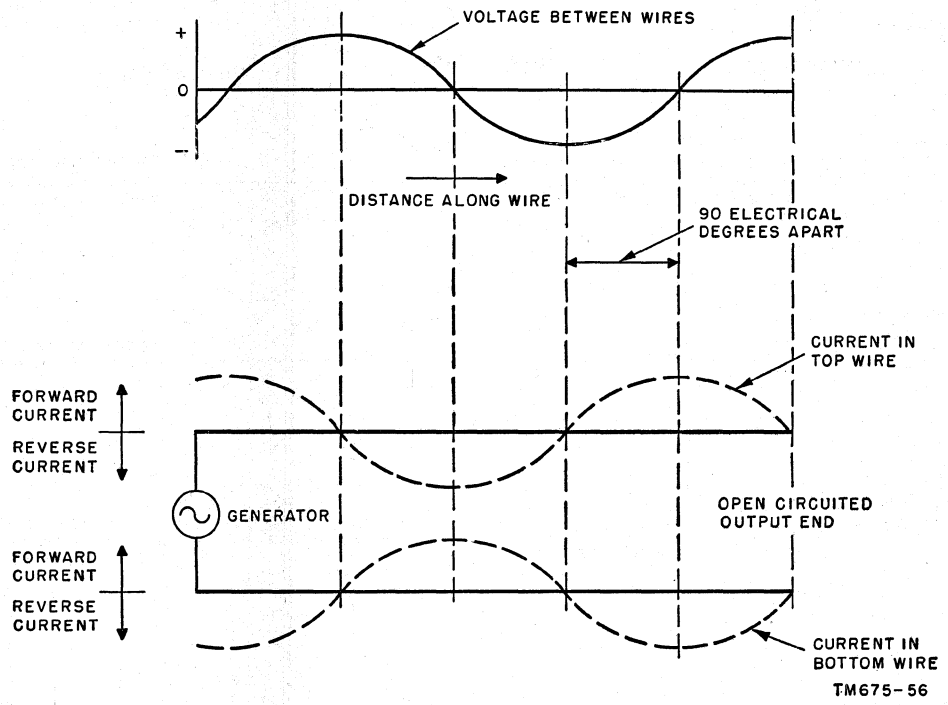


Figure 56. Reflection causes out-of-phase conditions.

ted along the line, the curve of voltage values forms a standing wave of voltage. Actually, the voltage at each point (except the zero voltage points) is alternating. However, the *rms* (root mean square) value of the alternation at any point remains constant. An a-c voltmeter, then, reads a series of *rms* values along the line. When these *rms* values are plotted, the curve represents a wave which appears to be stationary, or standing, on the line. Figure 57 shows a series of different positions of the generated (incident) and reflected waves which are added (heavy lines) to show how the standing waves are produced. Whenever two sine waves are added together, the sum is a sine wave. Therefore, the standing wave is a sine wave.

48. Standing Waves of Current

A condition similar to that of the voltage waves exists for the current, except that current waves are reflected *with a reversal of polarity*. The incident and reflected current waves combine to produce standing waves of current. The maximum values of the standing waves of current are 90°, one quarter-wavelength, offset along the line from the maximum values of the voltage standing waves (fig. 58).

49. Reflections on a Short-Circuited Line

The voltage and current relations on a short-circuited or closed-end line are shown in figure 59. Since a short circuit is a condition of zero impedance, the current at the closed end of the line is maximum and the voltage is minimum. Reflection occurs in the closed line for the same reason that it occurs in the open-end line; that is, none of the energy in the initial wave is absorbed by the load. The high current in the short circuit represents energy which the zero impedance load cannot absorb. The only place this current can go is back over the line; consequently, a wave of current is reflected from the load toward the sending end of the line.

50. Representation of Standing Waves

Standing waves of voltage can be measured with an r-f voltage indicator. As shown in figure 57, high-voltage and low-voltage points are found. The minimum points are called nodes and the maximum points are called antinodes. Theoretically, on a line with no losses, the minimum voltage (or current) is zero. However, there are losses in any real line, and current must flow to supply the energy lost. Conse-

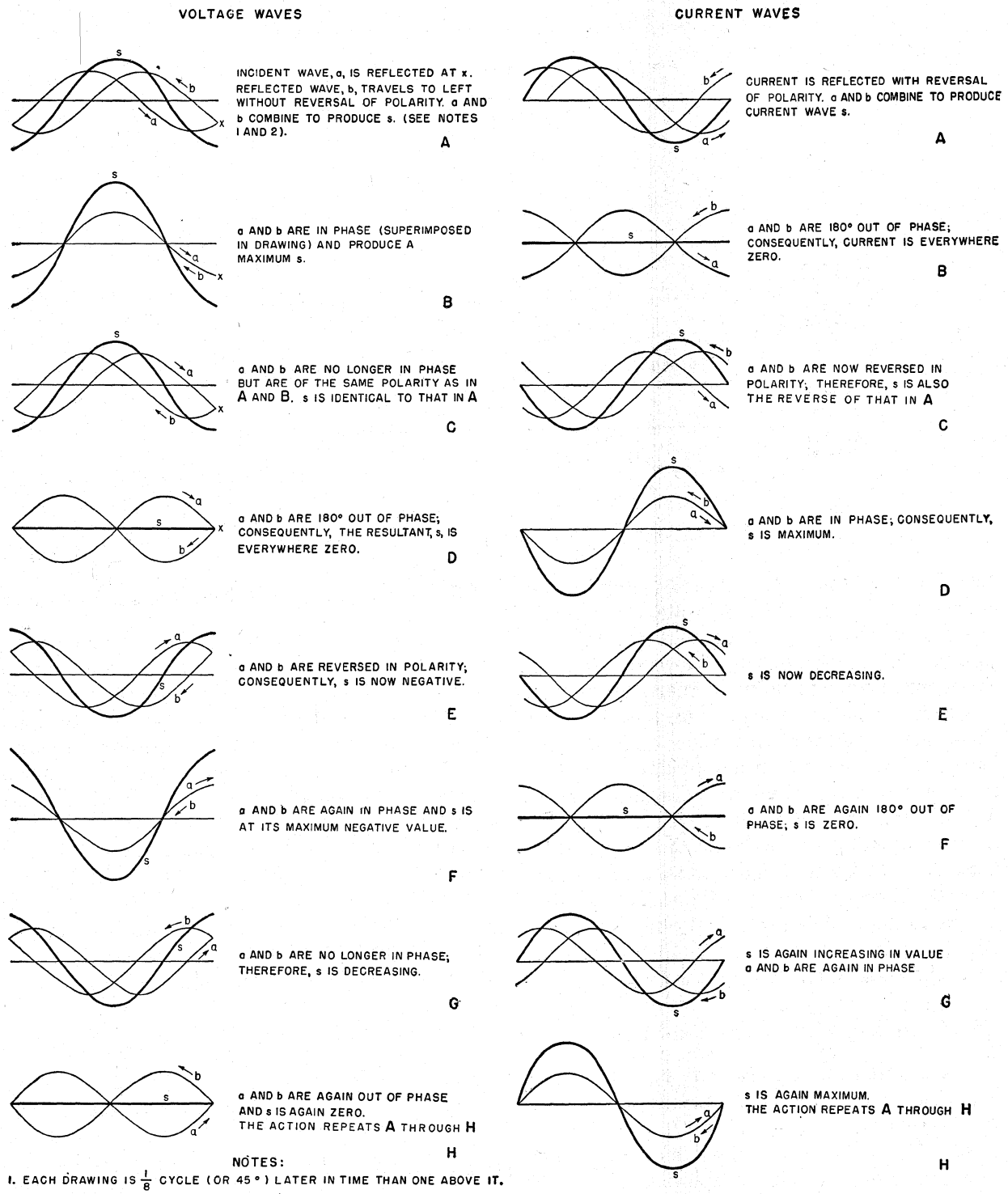

VOLTAGE WAVES
CURRENT WAVES
INCIDENT WAVE, a, IS REFLECTED AT x. REFLECTED WAVE, b, TRAVELS TO LEFT WITHOUT REVERSAL OF POLARITY. a AND b COMBINE TO PRODUCE s. (SEE NOTES 1 AND 2).
A
CURRENT IS REFLECTED WITH REVERSAL OF POLARITY. a AND b COMBINE TO PRODUCE CURRENT WAVE s.
A
a AND b ARE IN PHASE (SUPERIMPOSED IN DRAWING) AND PRODUCE A MAXIMUM s.
B
a AND b ARE 180° OUT OF PHASE; CONSEQUENTLY, CURRENT IS EVERYWHERE ZERO.
B
a AND b ARE NO LONGER IN PHASE BUT ARE OF THE SAME POLARITY AS IN A AND B. s IS IDENTICAL TO THAT IN A
C
a AND b ARE NOW REVERSED IN POLARITY; THEREFORE, s IS ALSO THE REVERSE OF THAT IN A
C
a AND b ARE 180° OUT OF PHASE; CONSEQUENTLY, THE RESULTANT, s, IS EVERYWHERE ZERO.
D
a AND b ARE IN PHASE; CONSEQUENTLY, s IS MAXIMUM.
D
a AND b ARE REVERSED IN POLARITY; CONSEQUENTLY, s IS NOW NEGATIVE.
E
s IS NOW DECREASING.
E
a AND b ARE AGAIN IN PHASE AND s IS AT ITS MAXIMUM NEGATIVE VALUE.
F
a AND b ARE AGAIN 180° OUT OF PHASE; s IS ZERO.
F
a AND b ARE NO LONGER IN PHASE; THEREFORE, s IS DECREASING.
G
s IS AGAIN INCREASING IN VALUE
a AND b ARE AGAIN IN PHASE
G
a AND b ARE AGAIN OUT OF PHASE AND s IS AGAIN ZERO.
THE ACTION REPEATS A THROUGH H
H
s IS AGAIN MAXIMUM.
THE ACTION REPEATS A THROUGH H
H
NOTES:
1. EACH DRAWING IS 1/8 CYCLE (OR 45°) LATER IN TIME THAN ONE ABOVE IT.
2. s IS INSTANTANEOUS VALUE OF a AND b.
TM675-57

Figure 57. Incident and reflected waves combine to produce standing waves.

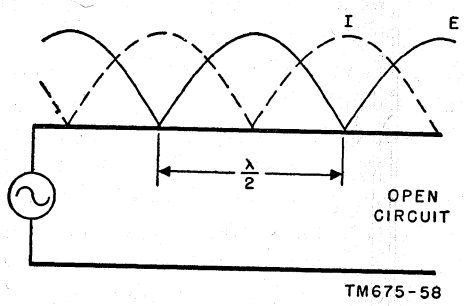


Figure 58. Voltage and current standing waves.

quently, no true zero points exist on a real line. Since indicators seldom are arranged to detect whether a standing wave is positive or negative, standing waves of voltage and current may be shown on one side of the base line (fig. 58). For convenience in drawing, most curves show the voltage between the conductors, and the current in one of the two conductors. In general, the top conductor (in the drawing) is used as a base line, or reference, for the waves (fig. 59).

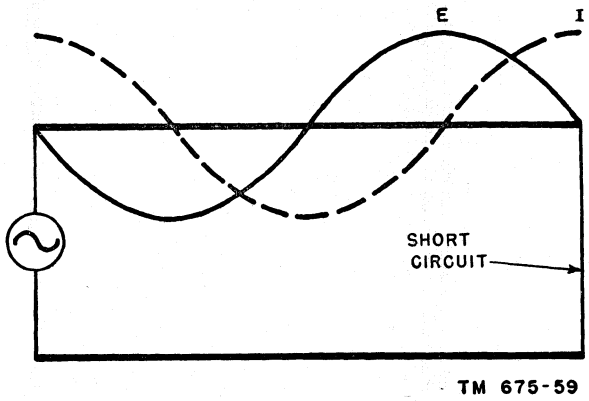


Figure 59. Standing waves on short-circuited line.

51. Comparison of Open- and Short-Circuited Lines

a. The voltage and current relations for open and closed lines are opposite to each other, as illustrated by the half-wave lines of figure 60.

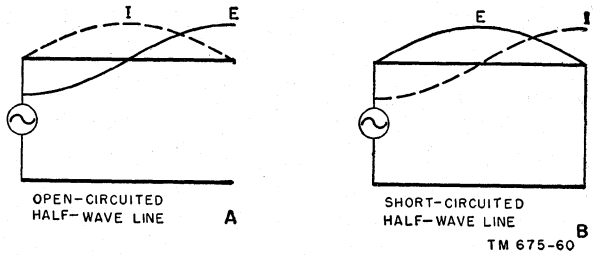


Figure 60. Voltage and current on open- and short-circuited half-wave lines.

b. The interrelation of open-circuited, short-circuited, and infinite-line conditions is shown in figure 61. Comparison of A and B shows that, for *any length* of line, the voltage and current relations in a closed line are opposite to those in an open line. The points of maximum and minimum voltage and current must be determined from the output end of the line, because reflection always *begins* at the output end. A line does not have to be of any particular length to produce standing waves. Standing waves occur on any line that is not terminated by a resistance equal to the characteristic impedance.

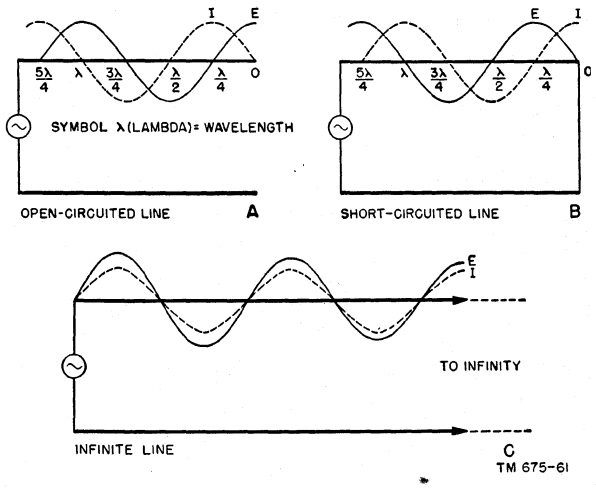


Figure 61. Comparison of line conditions.

Section IV. RESONANT LINES

52. Fundamental Idea

On a line which has standing waves of current and voltage, energy from the generator is surging back and forth along the line and most of this energy is in the electric and magnetic fields about the line. As waves of current travel along the line, the current points are sur-

rounded by magnetic fields. When the current is reduced to zero at the load end of an open-circuited line, the magnetic fields collapse and their energy cannot just disappear. Because there is no load to absorb it, this energy is transferred to the electric field. The increased electric field is evidenced by an increased voltage at the open end of the line. Energy given to the line by the generator can go only to two places, either to add energy to the electric and magnetic fields or to supply line losses, such as heat and radiation. If there were no losses whatever in the line, the generator could be removed and the line inductance and capacitance would discharge through one another in an *oscillatory self-sustained manner.* From a-c theory (TM 11–681) we know that, in the theoretical *L–C* circuit without resistance, this discharge of inductance and capacitance through one another would continue indefinitely, once started, even with the applied voltage removed. With line resistance and leakage of appreciable magnitudes, the line inductance and capacitance discharge through one another *against the line impedance,* and the oscillations decay with the removal of applied voltage. This corresponds to the similar decay of oscillations in an ordinary *L–C* circuit with resistance. We can now say that a transmission line can act as a resonant circuit.

53. Nonresonant Lines

It is well at this point to consider briefly what is meant by a nonresonant line. A nonresonant line may be defined simply as a line which has no standing waves of current and voltage. Such a line is either the theoretical infinite line or a real line which is terminated in a resistance equal to its characteristic impedance. The absence of standing waves is an indication that all of the energy reaching the load is absorbed by the load. C, figure 61, illustrates the voltage and current waves on a nonresonant line. The waves are traveling to the right toward the load end of the line.

54. Simple Definition of Resonant Line

a. A resonant line may be defined simply as a line on which there are standing waves of voltage and current. The standing waves indicate that the line is of finite length and that it is *not* terminated in a resistance equal to its characteristic impedance.

b. A line is sometimes said to be resonant at a certain frequency. This means simply that the line is designed to act as a resonant circuit at the specified frequency. Such a line may be either open-circuited or short-circuited at its load end, and it is (usually) cut to the correct multiple of a quarter-wavelength. If the electrical length of a line is not a multiple of a quarter-wavelength, the line will act as a reactance; that is, as a capacitance or an inductance.

55. Comparison of Line and L-C Circuit

A resonant transmission line assumes many of the characteristics of resonant L-C circuits composed of lumped inductance and capacitance. The more important characteristics that resonant lines have in common with the more familiar resonant circuits are given below.

a. SERIES RESONANCE.
- (1) Resonant rise of voltage across circuit elements.
- (2) Low impedance across the resonant circuit.

b. PARALLEL RESONANCE.
- (1) Voltage not in excess of applied voltage.
- (2) High impedance across the resonant circuit.

56. Diagrams

In studying applications of resonant transmission lines, careful examination should be made of figures 62 and 63. These illustrations show the relation of voltage, current, and impedance for various lengths of open-circuited and short-circuited transmission lines. The impedance which the generator sees for various lengths of line is shown directly above the generator on the charts. The curves above the letters of various height indicate the relative values of the impedance presented to the generator as it moves from right to left, and the circuit symbols indicate the equivalent (lumped) electrical circuits for an r-f transmission line of that particular length. The standing waves of voltage E and current I, whose ratio is the impedance Z, are shown above each line. These waves are shown on the chart as having the

same maximum heights and as going to zero at the minimum points. The last part of this statement actually would be the case only on a line having no loses. Figures 64 and 65 also show equivalent circuits of particular lengths of lines which have been taken from figures 62 and 63.

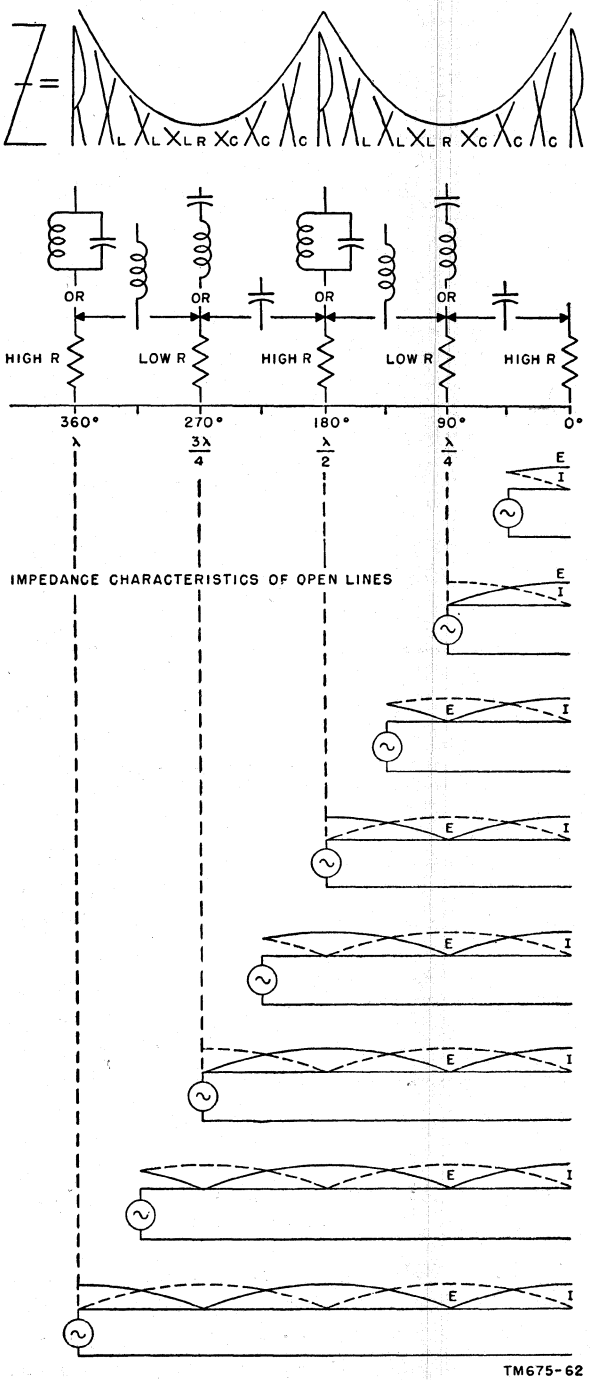


Figure 62. Characteristics of open-circuited lines.

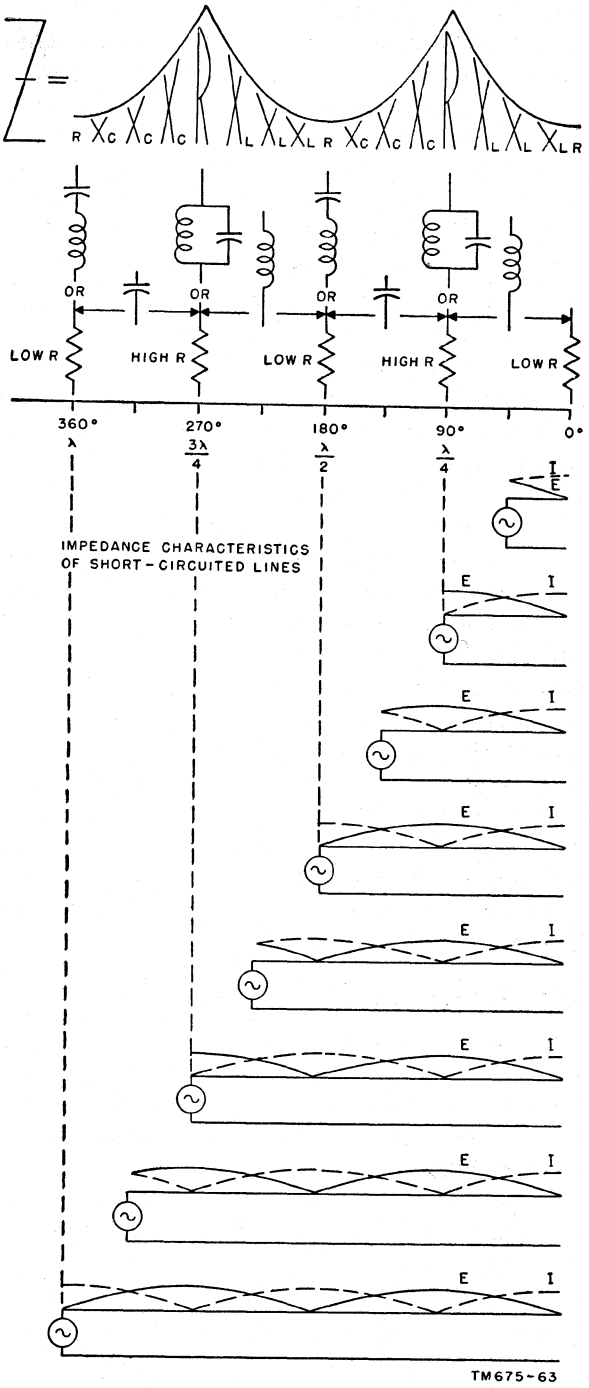


Figure 63. Characteristics of short-circuited lines.

57. Resonance in Open Lines

a. ODD QUARTER-WAVELENGTHS. The open-end line can be studied with the aid of figures 62 and 64. At all odd quarter-wave points ($\lambda/4$, $3\lambda/4$, etc.) measured from the output end, the

current is maximum and the impedance minimum. In addition, there is a resonant rise of voltage from the odd quarter-wave point toward the output end. Thus at all odd quarter-wave points the open-end transmission line is acting like a *series* resonant circuit. C, figure 64, shows the equivalent circuit for a quarter-wave line or any odd multiple thereof ($3\lambda/4$, $5\lambda/4$, etc.). The impedance is a very low resistance and would be zero if there were no losses in the line.

b. EVEN QUARTER-WAVELENGTHS. At all even quarter-wave points ($\lambda/2$, λ, $3\lambda/2$, etc.), figure 62 shows that the voltage is maximum. The voltage on an even quarter-wave line never exceeds the applied voltage. Comparison of the transmission line with an *L–C* resonant circuit demonstrates that at even quarter-wavelengths an open-end line acts like a parallel resonant circuit. F, figure 64, shows the equivalent circuit for a half-wave line or any even multiple of a quarter-wave line ($\lambda/2$, λ, $3\lambda/2$ etc.). The impedance is an extremely high resistance.

c. LINE AS INDUCTANCE OR CAPACITANCE. In addition to acting as *L-C* resonant circuits, resonant open-end lines also may act as nearly pure capacitances or inductances. Figure 62 shows that an open-end line less than a quarter-wavelength long acts as a capacitance (A, fig. 64) ; from $\lambda/4$ to $\lambda/2$ long, as an inductance (D, fig. 64) ; from $\lambda/2$ to $3\lambda/4$ long, as a capacitance; from $3\lambda/4$ to λ long, as an inductance, and so on. B, figure 64, shows that an eighth-wavelength open line acts as a capacitive *reactance* numerically equal to the characteristic impedance Z_0, and *E* shows that a three-eighths-wavelength open-line acts as an inductive reactance.

58. Resonance in Short-Circuited Lines

a. ODD QUARTER-WAVELENGTHS. The closed-end line can be studied with the aid of figures 63 and 65. At the odd quarter-wavelength the voltage is high, the current low, and the impedance high. Also, at no place in such a line does the voltage exceed the applied voltage. Since these conditions are similar to those in a parallel resonant circuit, the shorted transmission line of odd quarter-wavelengths acts like a parallel resonant circuit. C, figure 65, illustrates the equivalent circuit for this condition of a quarter-wavelength shorted line.

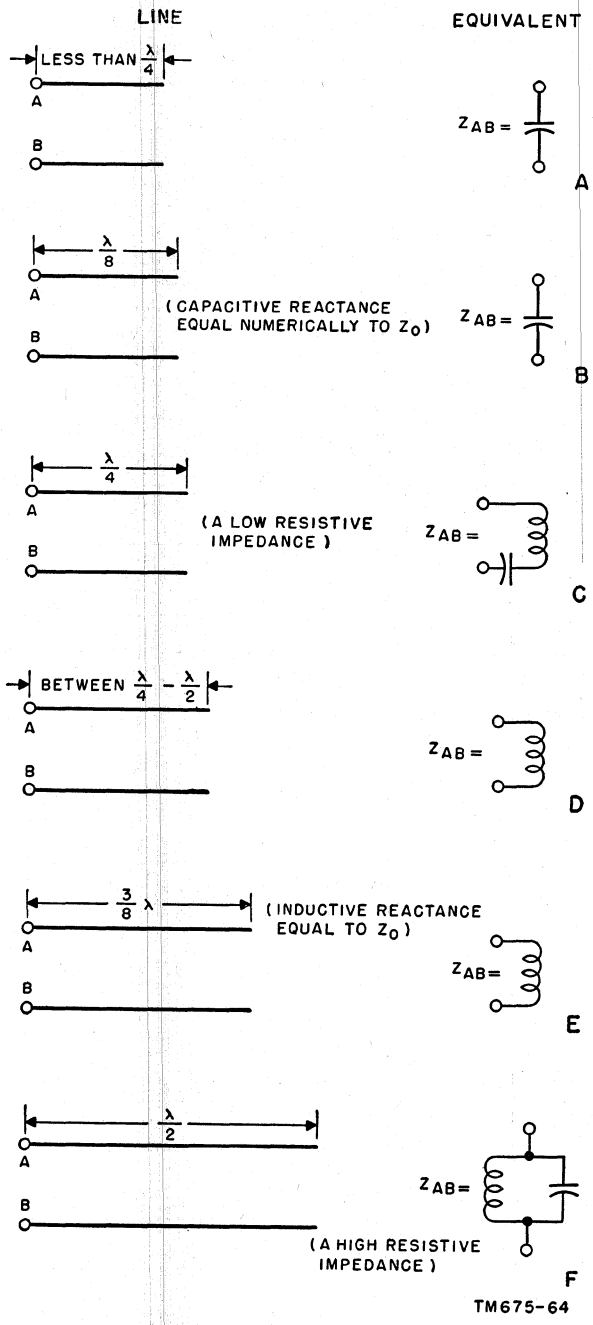


Figure 64. Open-circuited lines and corresponding lumped circuits.

b. EVEN QUARTER-WAVELENGTHS. At the even quarter-wave points, the voltage is minimum, the current maximum, and the impedance minimum. Since this action is similar to series resonance in an *L-C* circuit, a shorted transmis-

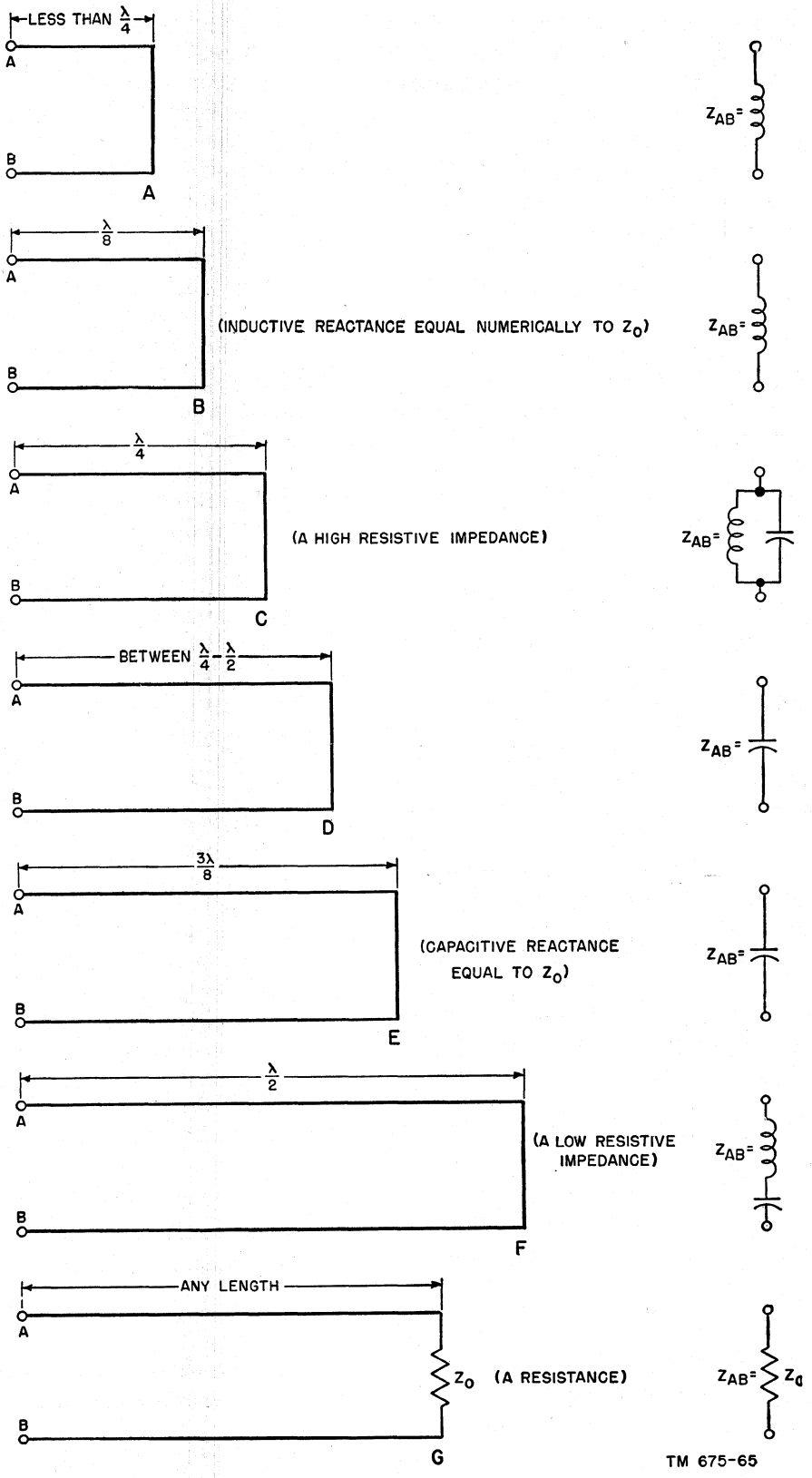


Figure 65. Short-circuited lines and corresponding lumped circuits.

sion line of even quarter-wavelengths acts like a series resonant circuit. Also, looking from any even quarter-wave point toward the output end, there is a resonant rise of voltage to the adjacent quarter-wave point. Consequently, the standing waves of voltage may be of considerably greater amplitude than the applied voltage. F, figure 65, illustrates the equivalent circuit for half-wavelength line.

c. LINE AS INDUCTANCE OR CAPACITANCE. Resonant closed-end lines, like the open-end lines, also act as nearly pure capacitances or inductances. Figure 63 shows that a closed-end line less than $\lambda/4$ acts as an inductance (A, fig. 65), from $\lambda/4$ to $\lambda/2$ long as a capacitance (D, fig. 65), and so on. B, figure 65, shows that a $\lambda/8$ line acts as an inductive reactance numerically equal to the characteristic impedance Z_o, and E shows that a $3\lambda/8$ line is equivalent to a capacitance. In G, figure 65, termination in Z_o is illustrated. The input impedance in this case is a resistance equal to Z_o.

59. Other Line Terminations

a. EFFECT OF TERMINATING IN RESISTANCE EQUAL TO Z_o.

(1) Any line, even if cut to a particular fraction of a wavelength, loses its resonant characteristics when it is terminated in a resistance equal to Z_o.

(2) Since a quarter-wavelength open-end line has a low E and a high I (fig. 62) there is a low impedance at AB in A, figure 66. Conversely, since a quarter-wavelength closed-end line has a high E and a low I (fig. 63), there is a very high impedance at AB in B, figure 66. Now if this same quarter-wavelength line is terminated in its characteristic impedance Z_o, it immediately becomes a nonresonant line and presents an impedance at AB equal toZ_o. In figure 67 no difference can be detected at terminals AB, whether the line is a quarter-wavelength line as in A or an infinite line as in B. In other words, the quarter-wavelength line in A has none of the characteristics of a quarter-wavelength resonant line (fig. 66) ; it is just a nonresonant line.

b. LINE TERMINATED IN A REACTANCE. A line terminated in a resistance equal to its characteristic impedance has no reflections present. However, if a line is terminated in a *reactance* of *any* value, standing waves are *not* eliminated. A, figure 68, shows the standing waves on a line terminated in a capacitive reactance equal to the characteristic impedance. B, figure 68, shows the standing waves on a line terminated in an inductive reactance equal to the characteristic impedance.

(1) With a capacitive reactance load (A, fig. 68) the first minimum point of voltage is closer than a quarter-wavelength to the output end of the line. Similarly, in B, figure 68, the first minimum point of voltage is more than a quarter-wavelength from the output end. With capacitive terminations the voltage and current distribution has essentially the same character as with the open circuit, except that the curves are shifted toward the output end of the line by an amount that increases as the capacitive *reactance* is reduced; that is, as the line approaches the closed-line condition.

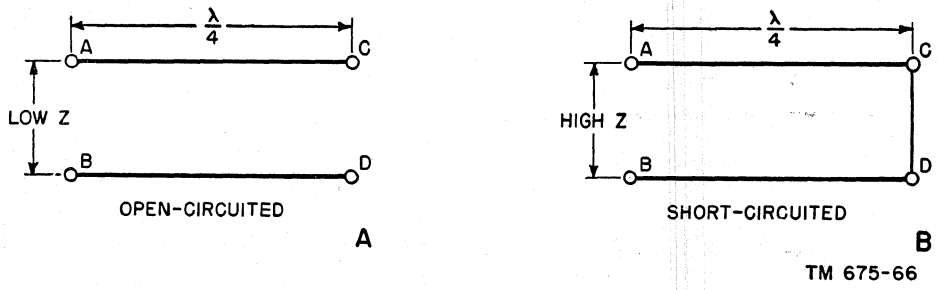


Figure 66. Quarter-wave lines.

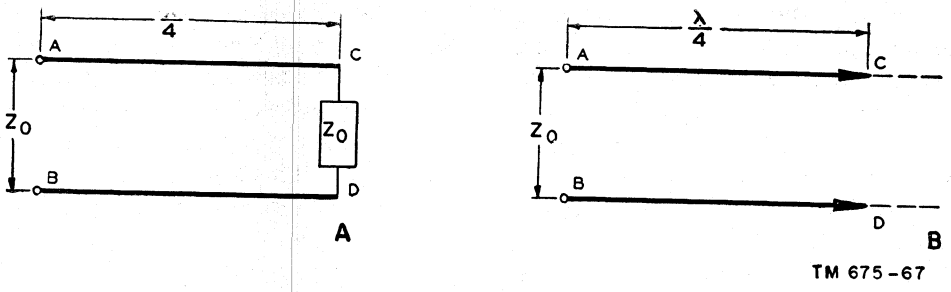


Figure 67. Quarter-wave line terminated in Z_o, A, behaves like infinite line, B.

(2) With inductive termination the voltage and current distribution has essentially the same character as with a short-circuited output end, except that the curves are shifted toward the output end by an amount that increases as the load reactance approaches infinity; that is, as the line approaches the open-line conditions.

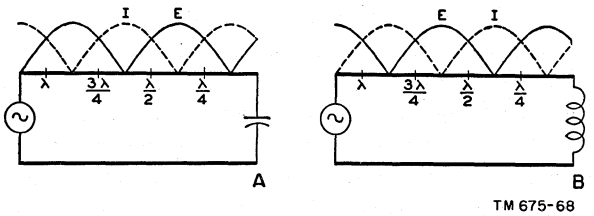


Figure 68. Standing waves produced by terminating in reactances equal to Z_o.

Section V. TYPES OF LINE

60. General

Four general types of transmission line are—

a. The two-wire or parallel-conductor line (fig. 69).

b. The concentric (coaxial) line (fig. 70).

c. The twisted pair (fig. 71).

d. The shielded pair (fig. 72).

61. Construction of Open Two-Wire Line

a. An open two-wire transmission line consists of two parallel conductors which are maintained at a fixed distance by means of insulating spacers or spreaders at suitable intervals (fig. 69). This line is used because of its ease of construction, its economy, and its efficiency. In practical applications open-wire transmission lines are used for commercial power lines, telephone lines, and telegraph lines, and as the connecting links between an antenna and a transmitter or an antenna and a receiver.

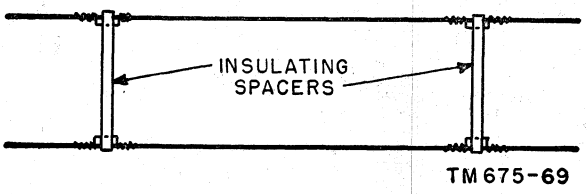


Figure 69. Construction of two-wire line.

b. In practice, the conductors of open-wire lines generally are spaced from 1 to 12 inches, depending on the impedance required and the diameter of the wire used.

c. The principal disadvantage of the open-wire transmission line is that it has high radiation losses. At extremely high frequencies these lines cannot be used in the vicinity of metallic objects because of the greatly increased loss which results.

62. Construction of Concentric Lines

a. The concentric or coaxial line has advantages which make it very practical for efficient operation at the high frequencies. It consists of a wire within and coaxial with a tubular outer conductor (fig. 70). In some cases the inner conductor also is tubular. The inner wire or conductor is insulated from the outer conductor by insulating spacers or beads at regular intervals. The spacers are made of pyrex, polystyrene, or some other material possessing good insulating qualities and low loss at high frequencies. Flexible coaxial cables also are made with the inner conductor consisting of flexible wire insulated from the outer conductor by a solid and continuous insulating material (fig. 5). Greater flexibility is gained by using a

metal braid as the outer conductor. The losses in this type of line are higher than those in a well-designed tubular type of concentric line.

b. The electromagnetic fields about a two-wire open line extend into space; consequently, radiation losses may be high and the lines are likely to pick up interference from external sources. In a coaxial line, however, the electromagnetic fields are confined to the space between the inner and outer conductors; that is, the coaxial line is a perfectly shielded line.

c. Owing to the perfect shielding, there is no radiation loss from a coaxial line and the line may be run close to, or attached to, metallic objects without causing any loss. In fact, one of the principle advantages of the coaxial line is that it may be run over the most convenient route, without increasing the losses over those due to attenuation within the line. The greatest disadvantage of a coaxial line is its relatively high attenuation, particularly at higher frequencies. The interior of tubular lines (fig. 70) must be kept dry to prevent excessive leakage between the conductors. To prevent the condensation of moisture within them, tubular lines may be filled with dry nitrogen at pressures ranging from 3 to 35 pounds per square inch. The nitrogen dries the interior of the line and the pressure is maintained to insure that leakage will be outward.

63. Construction of Twisted Pair

The twisted pair, as the name implies, consists of two insulated wires twisted to form a flexible line without the use of spacers (fig 71). It generally is used as an untuned (nonresonant) line for low-frequency transmission. It is not used at the higher frequencies, because of the high losses occurring in the rubber insulation. Its chief advantage is that it may be used over short distances where more efficient lines would not be feasible because of mechanical considerations.

64. Construction of Shielded Pair

a. The shielded pair (fig. 72) consists of two parallel conductors separated from each other and surrounded by an insulating dielectric material, such as the plastic copaline. The conductors are contained within a copper-braid tubing which acts as a shield for them. This assembly is covered with a rubber or flexible-composition coating to protect the line against moisture and friction. Outwardly, it looks much like an ordinary power cord for an electric motor.

b. The outstanding advantage of the shielded pair is that the two conductors are balanced to ground; that is, the capacitance between each conductor and ground is uniform along the entire length of the line and the wires are shielded against pick-up of stray fields. This balance is effected by the grounded shield which surrounds the conductors at a uniform spacing throughout their length. If radiation from an unshielded line is to be prevented, the current flow in each conductor must be equal in amplitude in order to set up equal and opposite magnetic fields which will cancel out. This condition may be obtained only if the unshielded line is well in

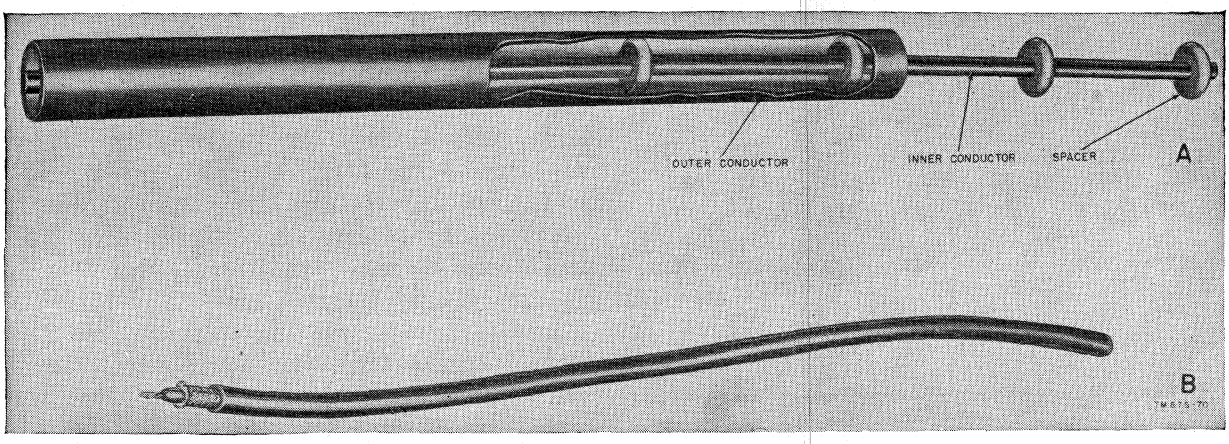


Figure 70. Construction of concentric (coaxial) line.

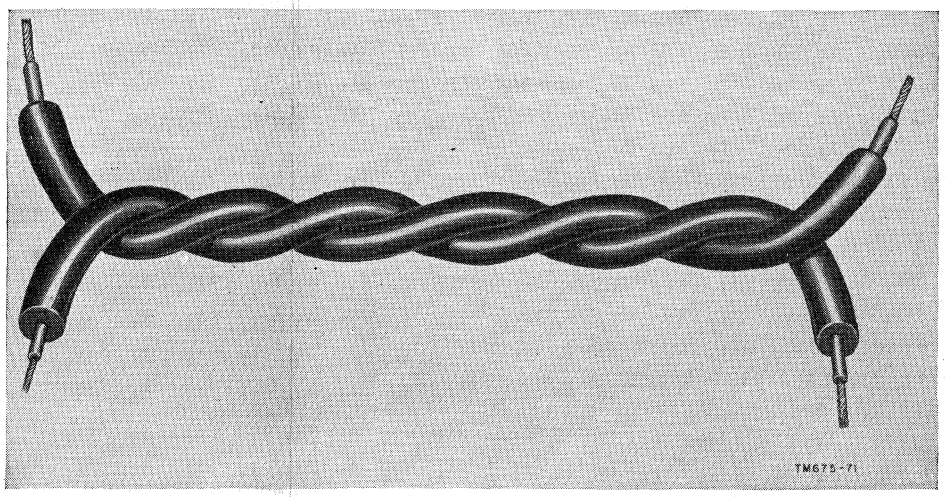


Figure 71. Twisted pair.

the clear of all obstructions, and if the distance between the wires is small. If an unshielded line runs near a grounded or conducting surface, one of the two wires is closer to the obstruction than the other. The capacitance between the closer conductor and the conducting surface is greater than the capacitance between the farther conductor and the conducting surface. These unequal capacitances act as unequal conducting paths for each half of the line, causing a greater current in the conductor closer to the nearby conducting surface. Because of these unequal line currents, radiation from the line is increased. The shielded line reduces these losses by maintaining balanced capacitances to ground.

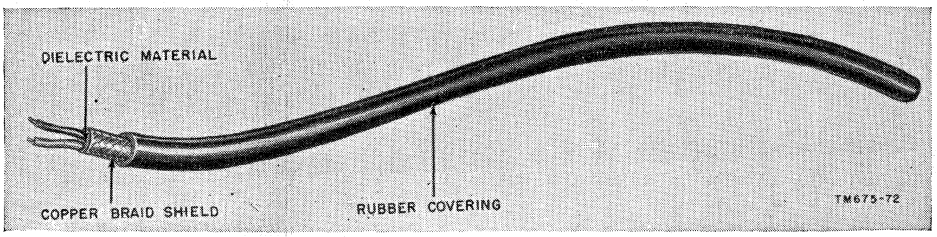


Figure 72. Shielded pair.

Section VI. LINE MEASUREMENTS

65. Characteristic Impedance and Wavelength

a. At radio frequencies the characteristic impedances, Z_o, of a well-designed transmission line may be considered as equal to the square root of L/C. A two-wire line having almost any desired characteristic impedance can be constructed in practical applications.

b. The approximate formula $Z_o = \sqrt{L/C}$ indicates that the characteristic impedance of an r-f line depends principally on the values of L and C in the line. An increase in the separation of the wires increases the inductance L and decreases the capacitance C. This effect takes place because the effective inductance is very low if two wires are closely spaced and carrying currents in opposite direction, and because the capacitance is low if the wires (plates of capacitor) are widely separated. The effect of increasing the spacing is to increase the characteristic impedance, since increasing L and reducing C increases the L/C ratio. A reduction in the diameter of the wires also increases the

characteristic impedance. This reduction affects the capacitance more than the inductance, for it is equivalent to decreasing the size of the plates in a capacitor in order to obtain lower capacitance. Any change in the dielectric material between the two wires also changes the characteristic impedance. Thus, if a change in dielectric material increases the capacitance between the wires, the characteristic impedance is reduced.

c. The characteristic impedance of a two-wire line with air as a dielectric may be obtained from the formula.

$$Z_o = 276 \log_{10} b/a$$

in which b is the spacing between the *centers* of the conductors and a is the radius of the conductor. This formula is sufficiently accurate at high frequencies where the characteristic impedance is practically a pure resistance.

Example: If two wires ¼ inch in diameter are spaced 2 inches apart, what is the characteristic impedance?

$Z_o = 276 \log_{10} b/a$
$b = 2$ inches
$a = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} = .125$ inch
$Z_o = 276 \log_{10} 2/.125 = 276 \log_{10} 16$

From a table of logarithms,

$Z_o = 276 \times 1.204 = 332.3$ ohms
$\log_{10} 16 = 1.204$

d. A few typical characteristic-impedance values obtainable with two-wire transmission lines are:

Wire gage (size of wire)	Spacing (inches)	Z_o (ohms)
18	2	560
10	2	440
18	3	610
10	3	480

66. Concentric Line

The characteristic impedance of a concentric line also varies with L and C. However, because of the difference in construction, L and C vary in a slightly different manner. The following formula must be used to calculate the characteristic impedance of a concentric line:

$$Z_o = 138 \log_{10} b/a$$

where b is the *inner* diameter of outer conductor and a is the *outer* diameter of inner conductor.

67. Impedance Charts

Two convenient graphs (fig. 73) make it possible to determine quickly the characteristic impedance of either a two-wire parallel or a concentric line when the ratio b/a is known.

a. Two-Wire Line. For example a two-wire line made of copper tubing of ¼-inch diameter and with a center-to-center spacing of 3 inches has a b/a ratio of $\frac{3}{1/8} = 24$. In A, figure 73, the value of 24 on the base line can be carried up to the slant line and thence over to the left to the vertical scale (see dotted lines). A value of approximately 375 ohms is obtained. A more exact value of 381 ohms can be calculated from the equation $Z_o = 276 \log_{10} b/a$.

b. Concentric Line. A concentric line having an inner-conductor diameter of 1/16 inch and an inner diameter for the outer conductor of ¼ inch has a b/a ratio of $\frac{1/4}{1/16} = 4$. In B, figure 73, a value of approximately 82 ohms can be calculated.

68. Elimination of Standing Waves

The difficulties of making high-frequency measurements are so great that it usually is more practical to calculate the characteristic impedance, Z_o, by means of the approximate formulas. The characteristic impedance of a line may be measured simply, and with sufficient accuracy for practical purposes, by the following method. Terminate the line with a *noninductive*, calibrated variable resistor. Adjust the resistance so that the standing waves are eliminated or reduced to the minimum possible magnitude. A suitable r-f indicator must be used to measure the standing waves (par. 70*d*). The resistance, read on the calibrated scale, which reduces the standing waves to zero, or to the minimum possible magnitude, is approximately equal to the characteristic impedance of the line. In most practical work it is not usually necessary to determine the characteristic impedance, since the only requirement

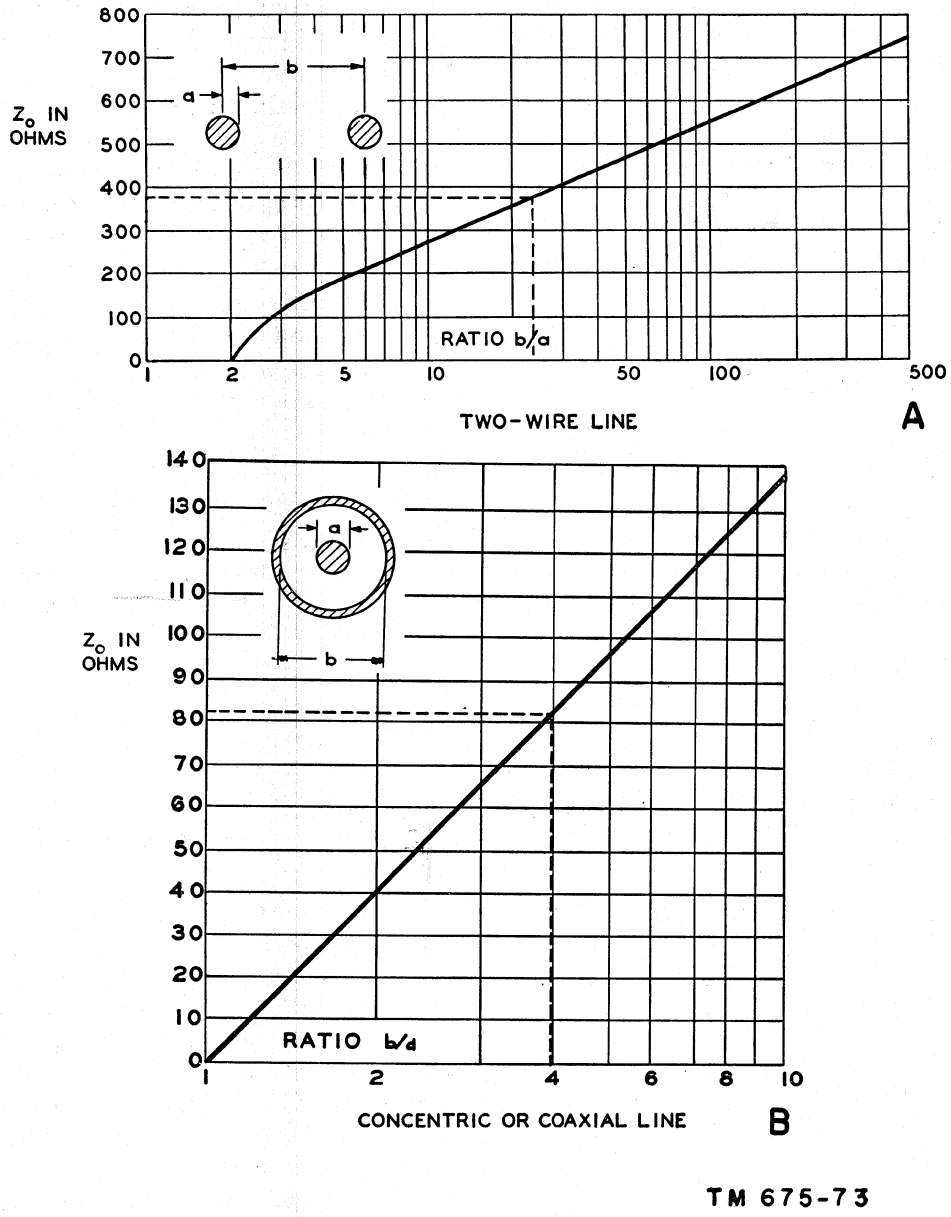


Figure 73. Characteristic impedance graphs.

is to eliminate the standing waves. To do this, vary the load at the output end of the line until the standing waves are reduced to the minimum possible magnitude.

69. Wavelength Measurements

a. Reflection always begins at the output end of a line. The distance *from the output end* to any maximum (or minimum) point is, therefore, a function of frequency and the line termination. It is convenient to measure these distances in terms of wavelength. The distance between successive maximum (or minimum) points of voltage (or current) is equal to one-half wavelength (fig. 74). One wavelength is *twice* the distance between successive maximum (or minimum) points. The distance corresponding to one wavelength of a wave in free space is given by the equation

$$\lambda = V/f = 300{,}000{,}000/f$$

where λ is wavelength in *meters*, V is the velocity in *meters*, and f is the frequency in *cycles*.

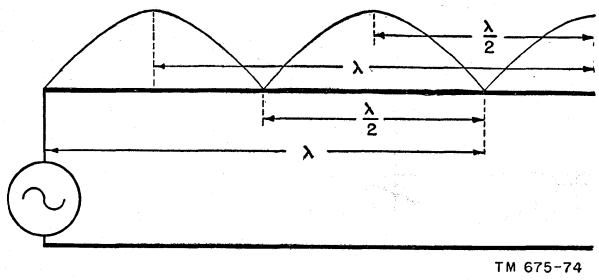


Figure 74. Standing waves as a measure of wavelength.

b. The velocity, V, of an electromagnetic wave varies with the medium through which the wave is passing. In mediums other than free space the velocity is always *less* than the velocity (300,000,000 meters/second) in free space. Referring to the equation for wavelength (*a* above), we see that if the velocity is reduced, the length of the wave must also be reduced. This means that in any real transmission line, the velocity is always less than 300,000,000 meters/second and, therefore, a wavelength on the line is *shorter* than the corresponding wavelength in free space. The electrical quarter-wavelength for various types of r-f line may be calculated from the following equation:

$$l = 246k/f$$

where l is the quarter-wavelength (measured in feet), f is the frequency in megacycles, and k is a multiplying factor. The factor k expresses the ratio of the actual velocity of the waves on a particular line to the velocity of the same waves in free space. Values of u for a few of the commonly used r-f lines are given in the following table.

Type of line	k
Parallel line	.975
Parallel tubing	.95
Concentric line (air insulated)	.85
Concentric line (rubber-insulated)	.56 to .65
Twisted pair	.56 to .65

70. Lecher Lines

Lecher lines is a term applied to lengths of parallel two-wire transmission lines which are used as tuned-circuit elements, or resonant lines, to measure wavelengths. Lecher lines usually are from one-quarter to five wavelengths and have a shorting bar (or bridge) which can be placed across the line and moved to any point along its length. In figure 75 the generator and the Lecher line are capacitively coupled through capacitors of very low capacitance (*high reactance*), so that the line will be open at its sending end. To provide practically short-circuit conditions across the input terminals, the line is inductively coupled to the generator. The inductance should consist of no more than a single turn of wire. At higher frequencies a straight shorting bar across the input terminals will provide sufficient coupling to the generator and, at the same time, establish essentially short-circuit conditions across the input terminals of the line.

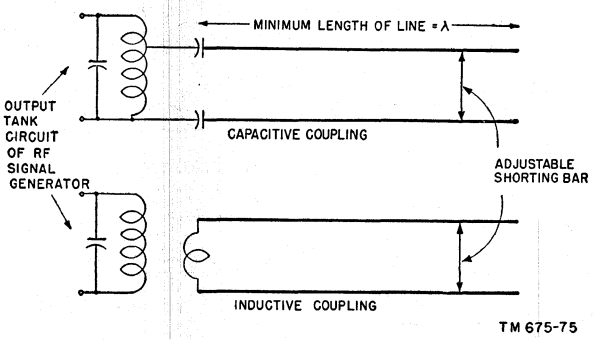


Figure 75. Methods of coupling lecher lines.

a. Consider the capacitively coupled Lecher line in figure 76. If the shorting bar is placed at the quarter-wave point (A, fig. 76), there is a reflection of current from the shorting bar and the standing waves of voltage and current are as shown. An ammeter, A, in the shorting bar indicates a high current. A plot of current readings in the wires at successive points from the bar toward the generator gives the current curve I. A similar plot of voltage readings gives the voltage curve E. We know that the impedance at any point in the line is given by the equation $Z = E/I$. At the input terminals (*CC*, A, fig. 76), I is zero and E is equal to the generator voltage; therefore, assuming that the resistance of the line is zero, the input impedance is

$$Z = E/I = E/O = \text{infinity}$$

This indicates that the shorted quarter-wave line acts as a parallel resonant circuit. In a real line there is, of course, a certain amount of

resistance and the actual input impedance is less than infinity. However, if the line resistance is reduced to the minimum possible value, the input impedance will be very high.

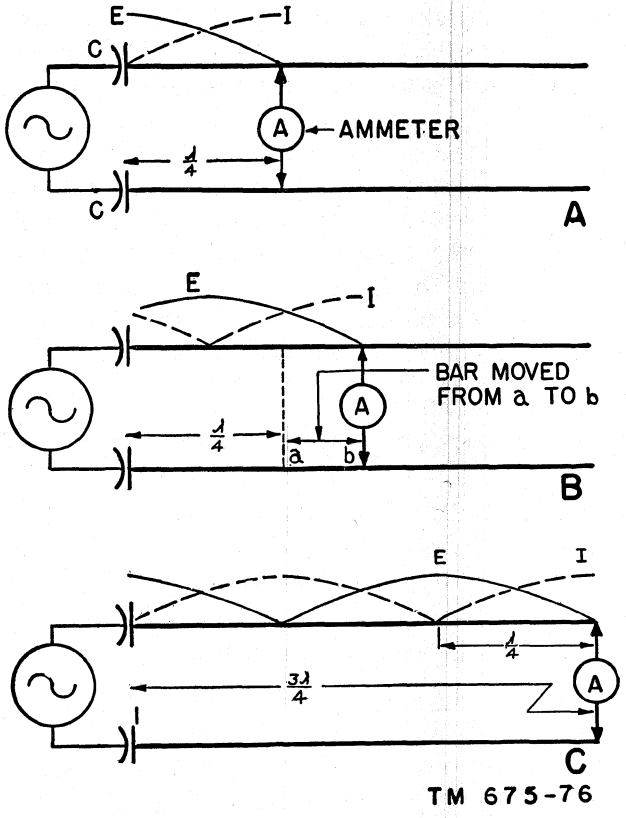


Figure 76. Variation of standing waves as shorting bar is moved.

b. If the shorting bar is moved from the quarter-wave position, there is little change in the current through it. However, there is a change in the standing waves (B, fig. 76), and the line is no longer acting as a parallel resonant circuit. Increased current from the generator into the line indicates that the input impedance is lower.

c. If the shorting bar is placed at the three-quarter-wave point (C, fig. 76) the voltage and current at the input terminals are identical to the voltage and current at the input terminals of the quarter-wave line (A, fig. 76). This assumes that there is no resistance in the line. With resistance in the line the current into the 3 $\lambda/4$ line would be somewhat higher than the current into the $\lambda/4$ line. This shows that *the maximum possible input impedance is obtainable with a $\lambda/4$ line which is shorted at the receiving end.*

d. Lecher lines may be used to measure wavelengths by detecting the positions of the voltage (or current) maximum (or minimum) point along the line.

(1) Current values can be measured roughly by the use of a small loop or coil of wire which can be coupled inductively to the line. A meter across the loop will read a maximum value when the loop is adjacent to a current maximum point, a minimum value when the loop is adjacent to a current minimum point. One wavelength is *twice* the distance between successive maximum (or minimum) points on the line. A and B, figure 77, shows simple r-f current indicators which may be used in determining the maximum and minimum points. In B, the diode rectifies r-f currents to provide direct current for the d-c meter. It might be well to note here that a point of maximum current corresponds to a

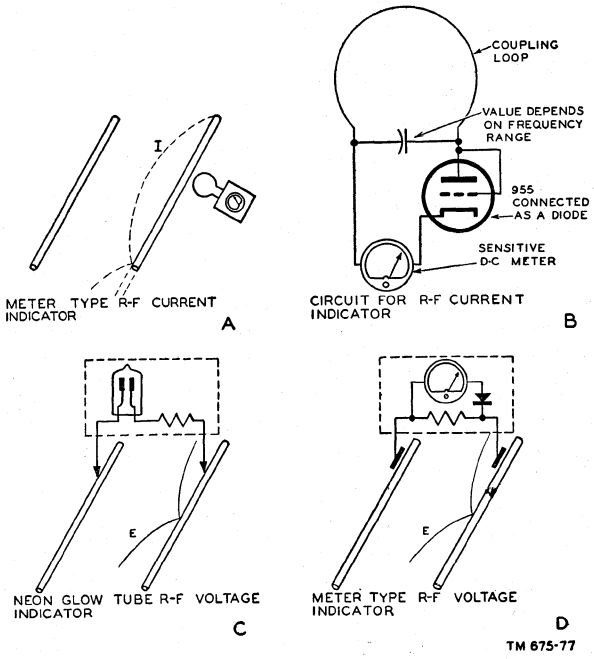


Figure 77. Simple r-f indicators.

point of minimum voltage, and a point of minimum current corresponds to a point of maximum voltage.

(2) It may be more convenient to use an r-f voltage indicator to locate the points of maximum and minimum voltage. C and D, figure 77, shows two simple types of r-f voltage indicators. The indicator in C consists of a neon bulb and a resistor connected directly across the line. In D the neon bulb is replaced by a meter and a crystal or or tube-type rectifier. The device is capacitively coupled to the line. The following method may be used to obtain very rough indications of the standing waves along a line. Hold an ordinary tubular-type, fluorescent light bulb by one end and bring the other end close to the line. If the line has standing waves the bulb will glow with varying brightness as it is moved along the line. Points of maximum brightness are points of maximum voltage on the line. For correct results the bulb must be held at a uniform distance from the line.

(3) The devices discussed in (2) above are simple and may be constructed easily. For accurate work well-designed indicators are available. For information on the theory and operation of these equipments refer to the technical manual for the particular item.

Section VII. PRACTICAL APPLICATIONS

71. Applications of Resonant Lines

a. GENERAL. R-f lines are used for many purposes other than the transmission of power from one point to another. For example, r-f lines are used as—

(1) Metallic insulators.
(2) Wave filters and r-f chokes.
(3) Reactors.
(4) Impedance-matching devices.
(5) Phase shifters and inverters.
(6) Oscillator frequency control.
(7) Line balance converters.

b. QUARTER-WAVE LINE AS A METALLIC INSULATOR. When a quarter-wave line is shorted at the output end and a voltage of the resonant frequency is applied, there are standing waves of voltage and current (A, fig. 78). At the short circuit the voltage is zero and the current is maximum. At the input end, however, the voltage is equal to the applied voltage and the current into the line is almost zero. Looking from terminals *AB* toward the grounded metal plate, current from the transmission line is almost zero; therefore, the quarter-wave section is acting as an insulator between the line and the grounded plate. If the frequency carried by the transmission line differs from the resonant frequency of the quarter-wave section, the quarter-wave section no longer acts as an insulator. B, figure 78, shows a quarter-wave section of coaxial line which is "teed" into a coaxial line to support the line.

72. Quarter-wave Line as Filter

a. The characteristics of a quarter-wave line also permit its use as an efficient filter or suppressor of even harmonics. Suppose that a transmitter is operating on a frequency of 5 megacycles per second and complaints are received that the transmitter is causing excessive interference on 10 and 20 megacycles. Along with other means of eliminating radiation at these even harmonic frequencies, a resonant transmission line may be tried as a harmonic suppressor.

b. C, figure 65, shows that a quarter-wave line shorted at the output end offers maximum impedance to the fundamental frequency, in this case 5 megacycles. However, at a frequency twice the fundamental frequency the line is a half-wave line and at a frequency four times the fundamental frequency the line becomes a full-wave line. F, figure 65, shows that a half- or full-wave line, which is shorted at the output end, offers zero impedance to this frequency. Therefore, the radiation of even harmonics from the transmitting antenna can be eliminated almost completely by means of

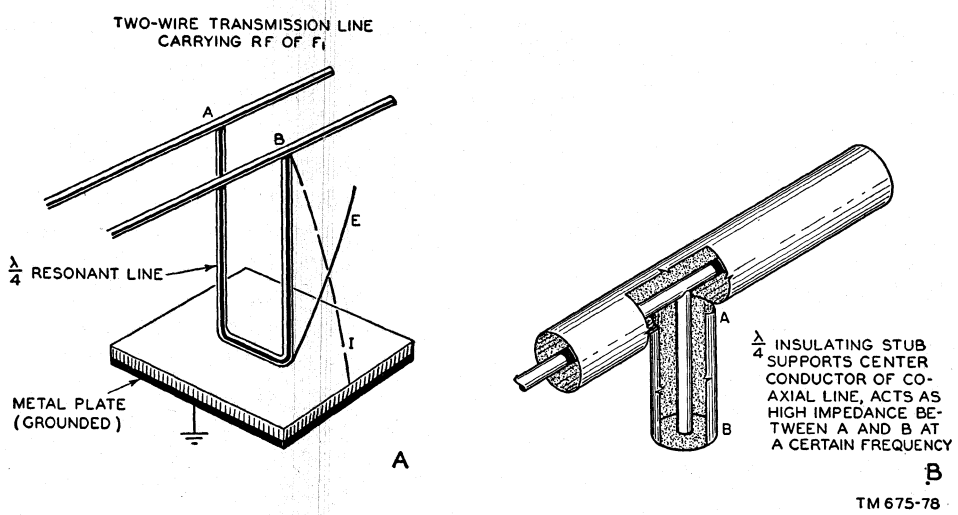


Figure 78. Quarter-wave lines as insulators.

the circuit shown in figure A, 79. The resonant line filter is a quarter-wave at 5 megacycles and offers almost infinite impedance to this frequency. At the second harmonic, 10 megacycles, the line is a half-wave and offers zero impedance, thus shorting this frequency to ground. At 20 megacycles, the filter line is a full-wave line, and again offers zero impedance, effectively grounding all 20-megacycles energy. The quarter-wave filter may be inserted anywhere along the nonresonant transmission line (B, fig. 79) with similar effect.

c. Both open and closed quarter-wave resonant lines may be used as wave filters. C, figure 79, shows how more than one line filter may be connected between a transmitter and an antenna to eliminate the radiation of undesired frequencies. In this case, a quarter-wave filter *B* which is open at the output end is placed in series with the transmission line. Reference to figure 62 shows that a quarter-wave line, which is open at its output end, offers low impedance to the fundamental frequency. At each odd harmonic such a line is an odd multiple of a quarter-wave and, therefore, offers little impedance to the fundamental frequency. At each odd harmonic such a line is an odd multiple of a quarter-wave and therefore offers little impedance to the odd harmonics. Thus, the quarter-wave open filter line *B* in C, figure 79, passes the fundamental and odd harmonics along the line to the antenna coupling unit. At the even harmonics, however, the length of the open line becomes a half-wave, or some multiple of a half-wave, so that line *B* offers high impedance to the even harmonics and blocks their passage to the antenna coupling unit.

d. The way in which two quarter-wave lines, one an open-end line *B* and the other a closed-end line *A*, connected as in C, figure 79, assist each other in filtering out even harmonics may be understood from the tabulation below:

Quarter-wave Line	Connection	Fundamental	Harmonics			
			2d	3d	4th	5th
Open	Series	Low Z	High Z	Low Z	High Z	Low Z
Shorted	Shunt	High Z	Low Z	High Z	Low Z	High Z

e. Unfortunately, this method cannot be used to eliminate odd harmonics, because any attempt to eliminate the odd harmonics also results in loss of the fundamental frequency. For example, assume that line *C* of C, figure 79, is a quarter-wave at the third harmonic—namely, 15 megacycles. This frequency would be eliminated effectively before it could reach the antenna. However, the fundamental that is to be transmitted also would be greatly attenuated. If a line is a quarter-wave at 15 megacycles it is a twelfth-wave line at 5 megacycles. Examination of figure 62 shows that a twelfth-wave line would act as a capacitor and offer a fairly low impedance to 5 megacycles. Therefore, although 15-megacycle radiation would be suppressed, the desired carrier also would be suppressed considerably.

f. Other types of filters can be built for more than the elimination of even harmonics. In fact, such filters are designed to eliminate efficiently the radiation of an entire single side band of modulated carrier.

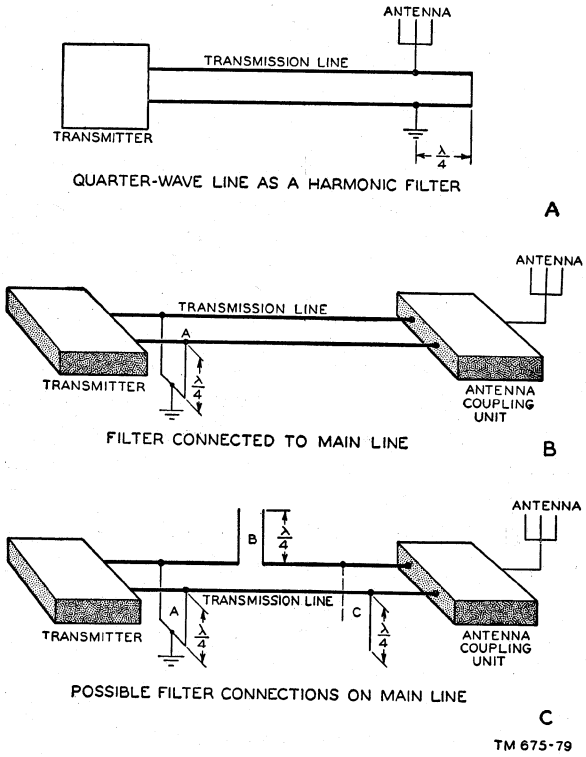


Figure 79. Quarter-wave lines as filters.

73. Transmission Line as Reactance

A transmission line may be used as a low-loss inductive or capacitive reactance. Figure 80 shows reactance variations for both the open and the closed output-end conditions of a line.

a. OPEN LINE. The open-end line, for example, acts as a capacitive reactance at lengths shorter than a quarter-wave. In fact, at an eighth-wavelength such a line has a capacitive reactance equal in magnitude to the characteristic impedance. Therefore, an open line less than a quarter-wavelength may be used as a capacitor, particularly when it is desired to avoid the use of actual capacitors in open-air installations. At the exact quarter-wavelength, or odd multiples thereof, the open line has zero reactance. As the length of the line is increased beyond a quarter-wave, the line becomes equivalent to an inductor, causing the current to lag. Finally, at a half-wave or multiple half-wavelength the open-end line has an infinite reactance.

b. SHORTED LINE. A closed-end line, on the other hand, acts as an inductive reactance at lengths shorter than a quarter-wavelength. At an eighth-wave it has an inductive reactance numerically equal to its characteristic impedance, and at exactly a quarter-wave, or odd multiple thereof, it has infinite reactance. Between a quarter-wavelength and a half-wavelength the line becomes equivalent to a capacitor, and at exactly a half-wavelength, or multiple thereof, the line has zero reactance. Circuit diagrams for all of the foregoing cases are shown in figures 64 and 65.

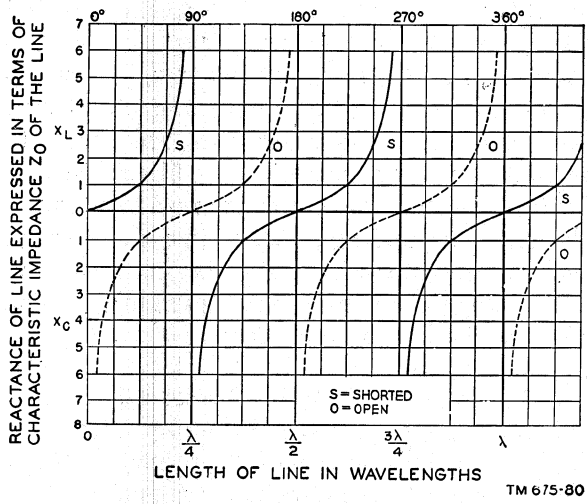


Figure 80. Transmission line reactance graph.

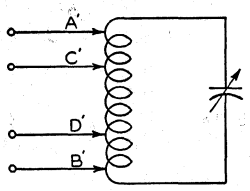
74. Line Sections for Impedance-matching

a. GENERAL. A parallel resonant circuit may be used to obtain an impedance match. For example, if a voltage of the resonant frequency is applied across points *A′* and *B′* in A, figure 81, the parallel resonant circuit offers maximum impedance to the line and, since the circuit is resonant, the impedance is the equivalent of a pure resistance. However, if the line voltage is applied across points *C′* and *D′*, the *L–C* circuit remains approximately in resonance, but the impedance offered to the line is considerably lower. By proper choice of the points *C′* and

D', the circuit may be matched to the exact line impedance, and the requirements for maximum power transfer met thereby.

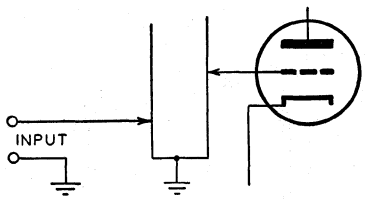
b. USE OF LINE.

(1) A transmission line also may be used for impedance-matching purposes (B, fig. 81). For example, it is desired to couple a resonant line to a nonresonant line. In order to be nonresonant, a line must be terminated in its characteristic impedance, and the terminating impedance should be approximately a pure resistance. The impedance of a shorted quarter-wave resonant section is zero at the shorting bar, increasing at points away from the shorting bar. Now, in order to adjust the section so that impedance across points *C* and *D* can be made equal to the characteristic impedance of the resonant line, it is necessary only to increase the distance between the bar and the points *CD* to increase the impedance, and to decrease this spacing to decrease the impedances. The impedance offered to the nonresonant line is resistive since the tuned section is resonant. It is assumed that the section is attached to the resonant line at *A* and *B* and, therefore, the shorting bar is adjusted to make a voltage maximum appear at *AB*. This is done before the connections *CD* are adjusted for the correct match to the nonresonant line.



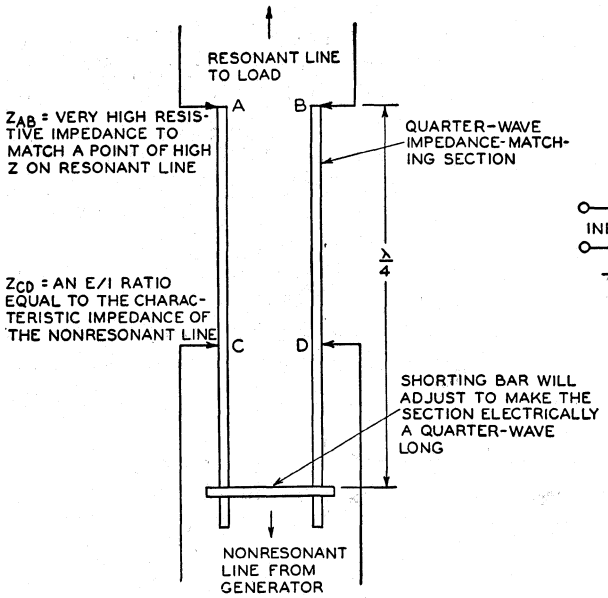
IMPEDANCE MATCHING WITH L-C RESONANT CIRCUIT

A



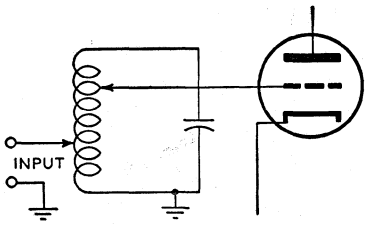
MATCHING TO GRID BY QUARTER-WAVE LINE

C


RESONANT LINE TO LOAD
A
B
Z_{AB} = VERY HIGH RESISTIVE IMPEDANCE TO MATCH A POINT OF HIGH Z ON RESONANT LINE
QUARTER-WAVE IMPEDANCE-MATCHING SECTION
$\frac{\lambda}{4}$
Z_{CD} = AN E/I RATIO EQUAL TO THE CHARACTERISTIC IMPEDANCE OF THE NONRESONANT LINE
C
D
SHORTING BAR WILL ADJUST TO MAKE THE SECTION ELECTRICALLY A QUARTER-WAVE LONG
NONRESONANT LINE FROM GENERATOR

IMPEDANCE MATCHING WITH RESONANT QUARTER-WAVE LINE

B



MATCHING TO GRID BY L-C RESONANT CIRCUIT

D

TM 675-81

Figure 81. Impedance matching.

(2) A second example of the use of a shorted quarter-wave section as impedance-matching device, or transformer as it is sometimes called, is shown in C, figure 81. Here a relatively low impedance input is transformed to a high impedance to match the high input impedance to a grid. The corresponding lumped circuit is shown in *D*.

(3) A half-wave section of line shorted at both ends is also used as an impedance-matching device, particularly in antenna-coupling problems. For example, figure 82 shows a half-wave section excited at *AB* and having resonant current and voltage values as shown by the curves labeled *E* and *I*.

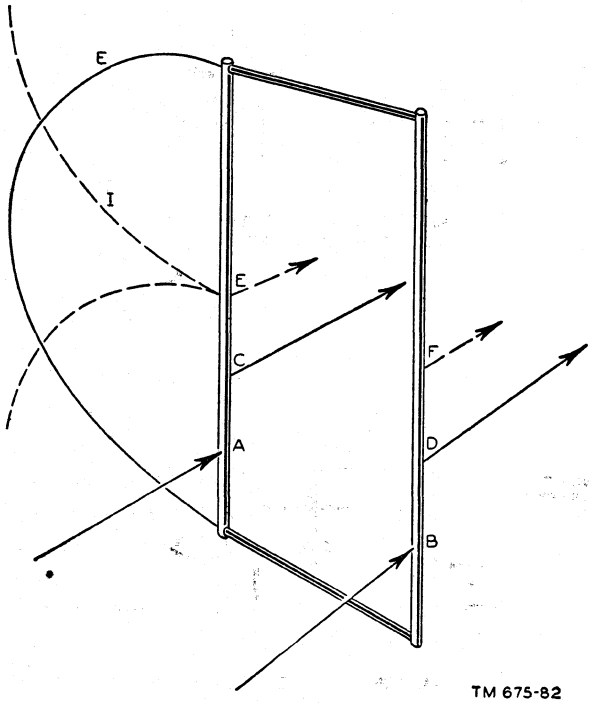


Figure 82. Shorted half-wave matching section.

The generator or input to *AB* sees an impedance equal to the *E/I* ratio at that point, or Z_{ab}, and the load or output looking into *CD* sees a larger *E/I* ratio and, hence, a larger impedance Z_{cd}. The greatest impedance will be obtained at *EF* where the voltage is highest and the current lowest. Conversely, the lowest impedance points will be at the shorting bars where the current is high and the voltage low. Since the upper half of the half-wave section, or half-wave frame as it is sometimes called, repeats the impedance of the lower half, there will always be two points on the frame which have the same impedance. There will be a difference, however, in the phase of the currents involved, the current on one half being 180° out of phase with that on the other half.

75. Lines as Impedance-Matching Devices

If a quarter-wave section is not shorted, but has a load impedance Z_r across the load end of the section, the input impedance is

$$Z_s = \frac{(Z_o)^2}{Z_r}$$

where Z_s is the input impedance, Z_o is the characteristic impedance, and Z_r is the impedance of the load. When Z_s and Z_r are known the equation can be rearranged to give the characteristic impedance. Then:

$$Z_o = \sqrt{Z_s Z_r}$$

Example: In figure 83 assume that line 1 has a characterictic impedance of 650 ohms and line 2 has a characteristic impedance of 400 ohms. The quarter-wave transformer provides a per-

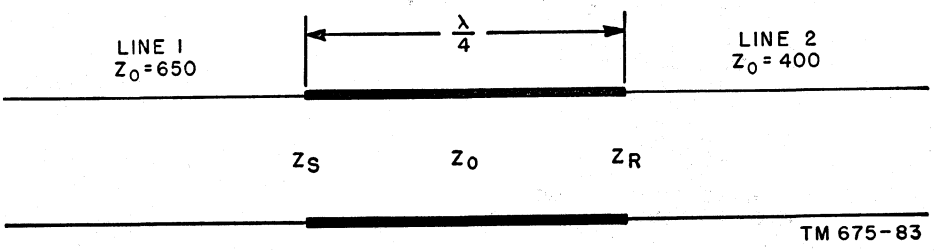


Figure 83. Quarter-wave transformer.

fect impedance match. What is the characteristic impedance of the quarter-wave transformer?

Solution: The 400-ohm characteristic impedance of line 2 is the load impedance, Z_r, of the quarter-wave transformer section. Since the transformer provides a perfect match, its *input impedance*, Z_s, must be equal to the characteristic impedance of line 1; therefore, $Z_s = 650$ ohms. Since we know Z_r and Z_s for the quarter-wave transformer, we can calculate the required characteristic impedance, Z_o, as follows:

$$Z_o = \sqrt{Z_s \times Z_r}$$

$$Z_o = \sqrt{650 \times 400}$$

$$Z_o = 510 \text{ ohms (approx)}$$

To check the work, we can rearrange the formula; then

$$Z_o^2 = Z_s Z_r$$

and,

$$Z_s = \frac{Z_o^2}{Z_r}$$

$$Z_s = 510^2/400 = 650 \text{ ohms}$$

Similarly, a 550-ohm line could be matched to a 70-ohm antenna as shown in figure 84. For the quarter-wave section $Z_o = \sqrt{550 \times 70} = 196$ ohms, approximately. In practice, matching sections of this nature, as well as those used to connect two transmission lines, are frequently referred to as quarter-wave transformers.

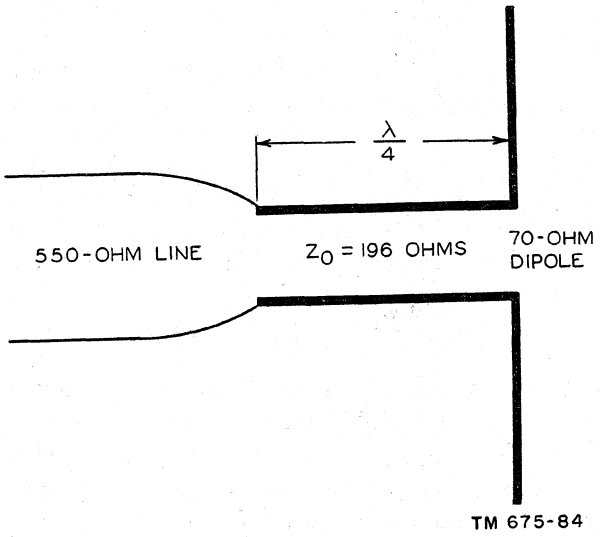


Figure 84. Quarter-wave transformer between line and antenna.

76. Half-wave Lines for Matching

If a load impedance, Z_r, is connected to the load end of a half-wave line (A, fig. 85), the input impedance, Z_{ab}, is equal to the load impedance Z_r, regardless of the characteristic impedance, Z_o. Because of this reflection of the load impedance to the input terminals, a half-wave line is said to act as a 1 to 1 transformer. When the impedance of a generator and a load are matched, a half-wave line, or any integral multiple thereof, may be used to interconnect them without disturbing the impedance match (B, fig. 85). If there is a mismatch between the impedances of a generator and its load, interconnecting them by a half-wave line, or any integral multiple thereof, will not affect the *mismatch* (C, fig. 85).

77. Stub Lines

a. A transmission line several wavelengths or more in length is used frequently to feed a load such as an antenna array. In order to eliminate standing waves, it is desirable to match the load or array to the line. A practical way of obtaining this match is by the use of a stub-line section connected across the main transmission line at a location near the load end. Although both open and shorted stubs may be used, a shorted stub generally is employed because of its lower radiation losses, and because it is easier to adjust and support.

b. The stub is located on the main line (A, fig. 86), at such a spacing from the load end that the main line, L_3, sees an impedance (looking into AB) which is for all practical purposes a pure resistance equal to the characteristic impedance of the main line. Reflection and standing waves are thereby eliminated from the main line, L_3. The impedance at AB will be made up of the impedances presented by L_1 and L_2 in parallel. L_1 is of such length that its input impedance at AB consists of a resistive component equal to Z_o and some reactive component. This reactive component, shown in B, figure 86, as X_c, is present because the load impedance is not equal to the characteristic impedance of L_1. The stub, which is normally less than a quarter-wavelength, has an input impedance that is almost a pure *reactance*. Therefore, the stub length can be adjusted, at points

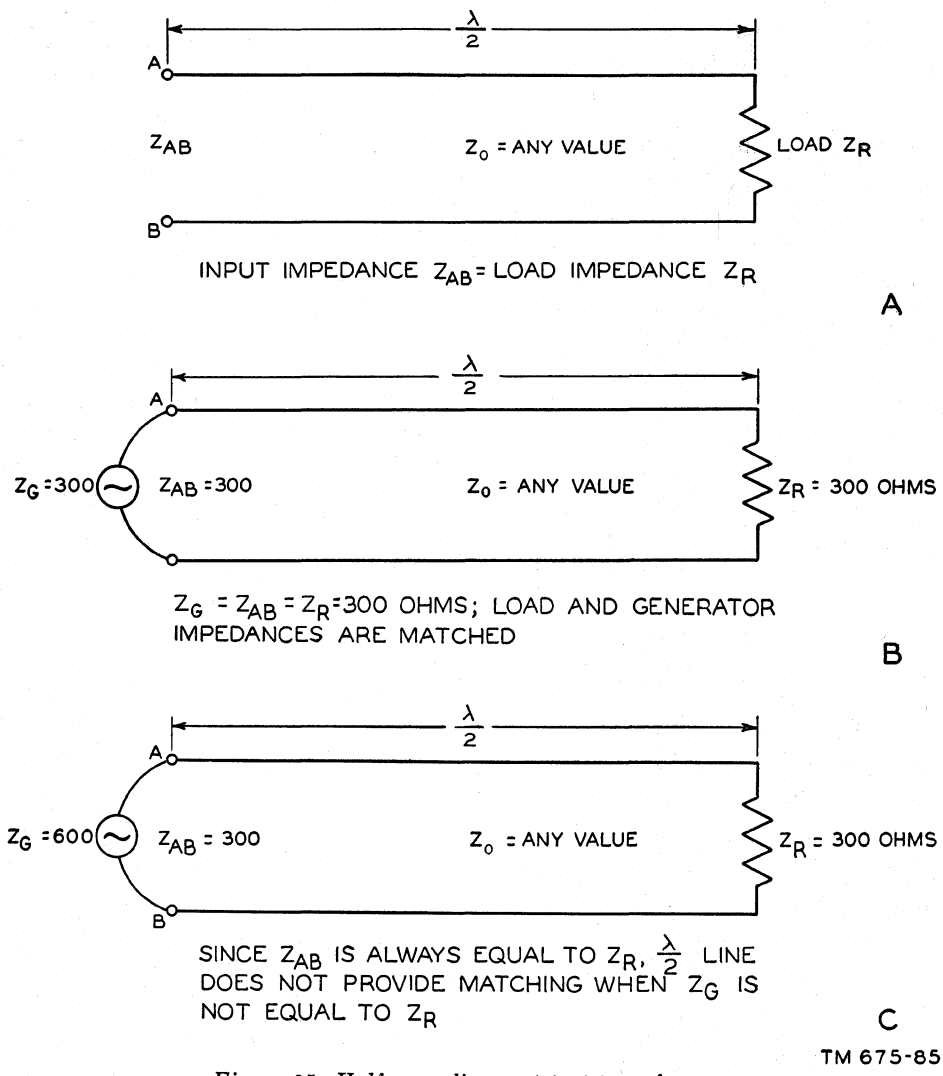


Figure 85. Half-wave line as 1 to 1 transformer.

C and D in A, figure 86, to produce at AB a reactive component, X_L, which cancels the reactive component, X_c, due to L_1. The result is that only a resistance equal to the characteristic impedance, Z_o, of L_3 remains across AB. B, figure 86, shows the equivalent circuit. It is assumed that the generator is matched to the line and that the line and generator can be represented by L_3 in B, figure 86.

c. The location of AB and CD can be found by a cut-and-try method, but the operation is rather tedious. It is difficult to find experimentally the length of L_1 which will present across AB a resistive component equal to Z_o. Also, after this point is found, any adjustment of L_2 affects the impedance across AB and the length of L_1 must be readjusted, and so on.

d. The following method enables a much faster, simpler, and systematic placement of the stub. By means of an r-f voltage indicator locate the voltage maximum point nearest the antenna. Similarly, at a distance of a quarter-wavelength from this maximum, locate the voltage minimum point. The standing wave ratio can then be determined:

$$\text{Standing wave ratio} = \frac{E_{max}}{E_{min}}$$

Use the graph of figure 87 to determine the stub location and stub length in wavelengths.

e. For example, assume that a voltage maximum of 200 and a voltage minimum of 50 are found by the use of an r-f voltmeter. Their ratio will be $E_{max}/E_{min} = 200/50 = 4$. From the graph it can be determined that the required

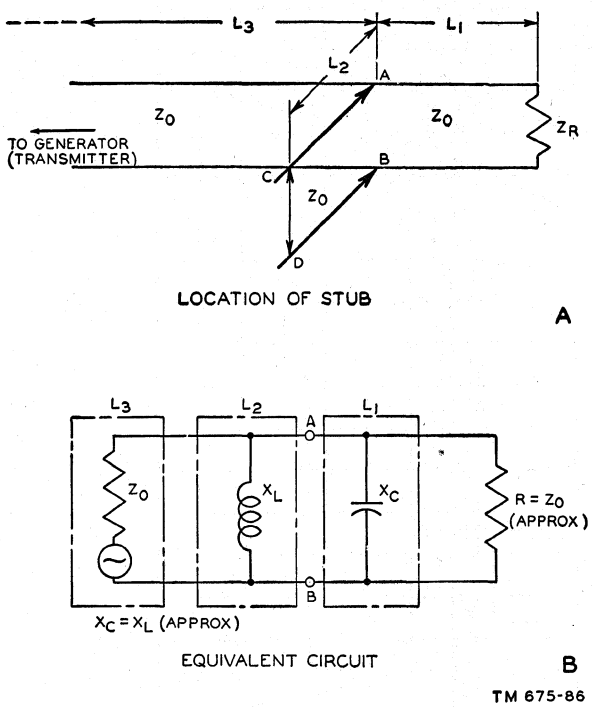


Figure 86. Shorted stub for matching.

stub location is .18 wavelength from the voltage maximum point. If a shorted stub is used, its length will be approximately .092 wavelength. On the other hand, an open stub will have a length of .156 wavelength. The stub location is from the voltage maximum toward the generator if the stub is short-circuited, and from the voltage maximum toward the load if the stub is open-circuited. All lengths shown on the graph of figure 87 are in wavelengths. Figure 88 shows the stub locations and the voltage waves on the line for both conditions.

f. Under some conditions it may be easier to locate the points of maximum and minimum current. These locations serve equally well for use with the graph, if it is noted that a current maximum coincides with a voltage minimum and a current minimum with a voltage maximum.

g. The principle of stub matching is applicable to coaxial lines as well as to parallel two-wire lines. However, on coaxial lines it is customary to use two stubs fixed in position along the line, each stub having a movable shorting plunger to vary the respective stub lengths. The two stubs are used to avoid the problem of moving the one stub along the line. The second stub adds capacitance or inductance to the line and thus varies the position of the standing waves. This produces the same effect on the first stub as if it were moving along the line. The two stubs are spaced either an eighth or a three-eighths-wavelength apart and near the load as shown in figure 89.

78. Lines as Phasing Devices

a. In many instances two or more antennas are operated simultaneously from the same transmitter, but with a phase difference between the two antenna currents. One means of providing the phase difference is by employing transmission lines of different lengths between the transmitter and the antennas. This method can be used because there is a progressive phase delay of 360° per wavelength along a non-resonant line. For example, if one line is an eighth-wavelength longer than the other, the phase difference is 45°. That is, a voltage or current maximum will arrive at the terminals of the longer line one-eighth cycle later than the corresponding maximum arrives at the terminals of the other. Figure 90 shows two antennas which have a 180° phase difference, because the line to antenna 2 is a half-wavelength longer than the line to antenna 1. The phase delay that occurs in a transmission line can be used only when no standing waves exist on the line.

b. A 1 to 1 ratio transformer can be used as a phase inverter (A, fig. 91). Since a half-wave line can act as a 1 to 1 transformer, it may also be used as a phase inverter. As shown in B, figure 91, the output voltage of a half-wave line is 180° out of phase with the input voltage.

79. Frequency Control

a. Quarter-wave, short-circuited transmission lines act as parallel resonant circuits, and may be substituted for the coil and capacitor circuits ordinarily used in oscillators.

b. The Q, or figure of merit, of a transmission line is given by the formula

$$Q = \frac{2\pi Z_o f}{R_c}$$

where R is the line resistance per unit length and c is the velocity of light in the same units

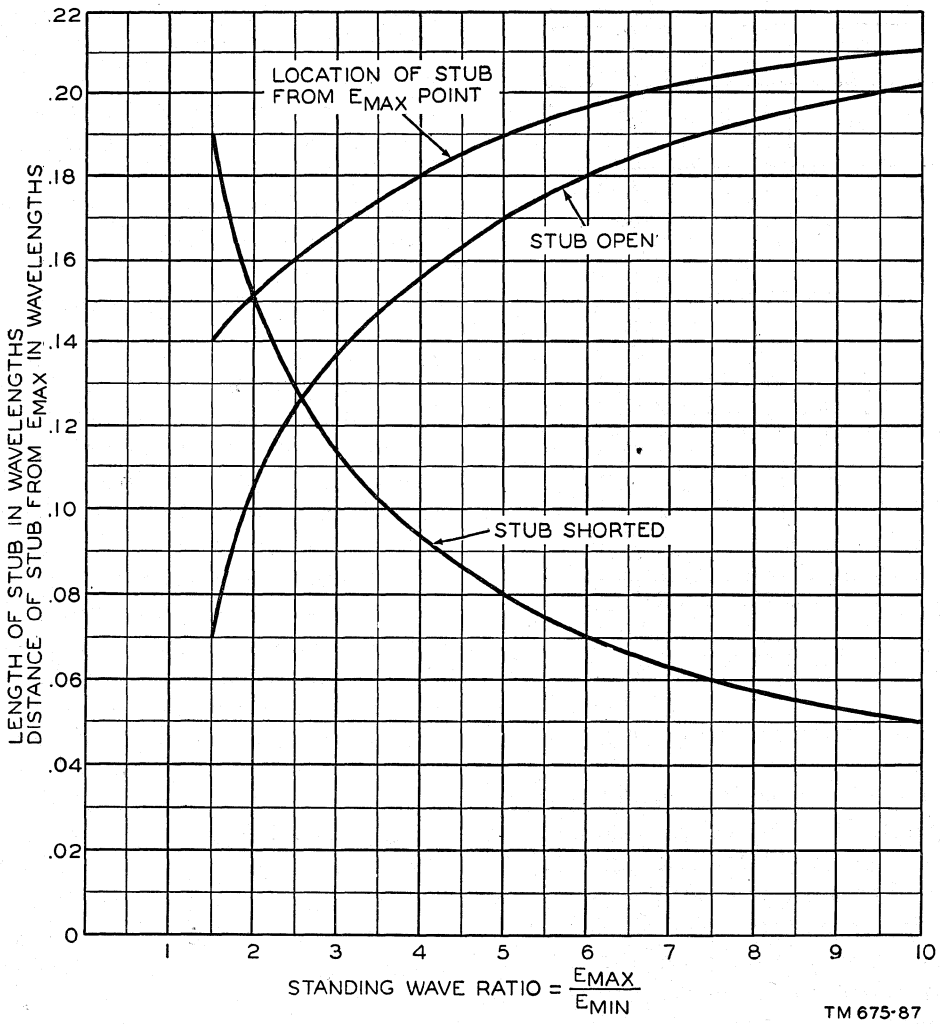


Figure 87. Graph for determining stub lengths and locations.

of length as R. In a well-designed r-f line R is extremely low; consequently, the Q of such a line is very high. The frequency stability of an oscillator is a function of the Q of its tuned circuits. Since the Q of a resonant line is much greater than the Q of ordinary coil- and capacitor-tuned circuits, a transmission line oscillator (fig. 92) can be designed to give frequency stability approaching that of crystal-controlled oscillators. The Q of a resonant line is affected by the line proportions. A quarter-wave resonant line provides a maximum Q when

$$b/a = 3.6$$

where b is the *inner* radius of the outer conductor and a is the outer radius of the inner conductor.

c. In a single-tube oscillator the concentric line is used frequently. For push-pull circuits, a balanced two-wire line or two concentric lines may be used. The concentric line has the added advantage that the inner conductor is shielded by the outer conductor, so that unwanted radiation is reduced to a minimum.

d. A simplified line oscillator circuit is shown in figure 92. This circuit may be tuned by means of the capacitor-shorting bar. It readily oscillates at the highest frequency of which the tube is capable.

80. Line Balance Converter

A coaxial line is *unbalanced* with respect to ground; an open-wire line or an antenna is *balanced* to ground. When a coaxial line is connected to an open-wire line or to an antenna, a balancing arrangement must be inserted at

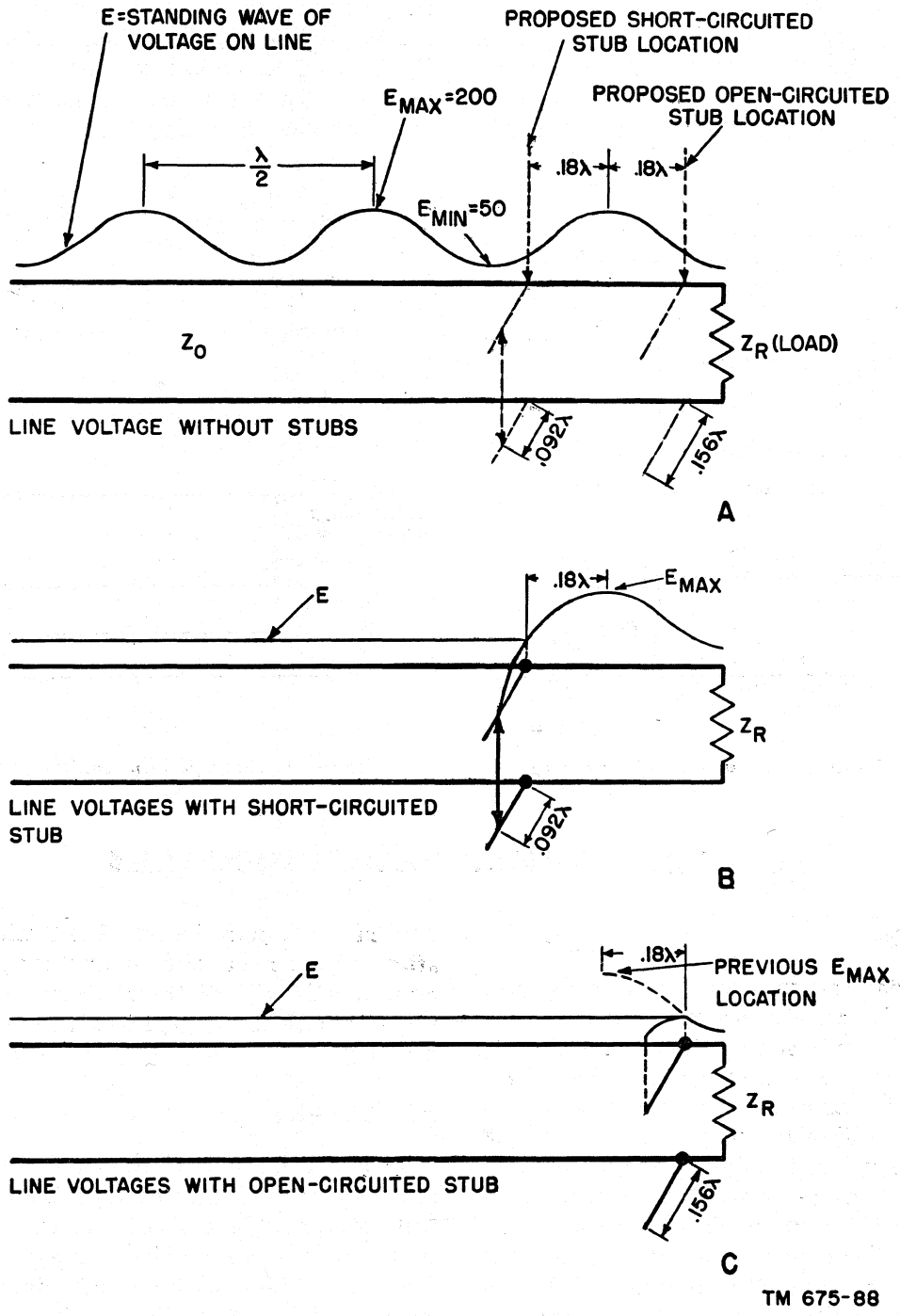


Figure 88. Single stubs for matching.

the junction of the balanced and unbalanced circuits. The balancing arrangement takes the form of a quarter-wavelength transformer. This may be in the form of a sleeve, open at one end and short-circuited at the other, placed over the coaxial line (A, fig. 93). The open end of the sleeve is flush with the end of the outer conductor of the coaxial line (*A* in B, fig. 93), and presents a very high impedance which insulates the conductor, *A*, from the outer surface of the sleeve and the line. The conductors of the two-wire line are connected as shown and, because of the high impedance between *A* and *B*, the voltage applied to them is balanced to ground. The analogous transformer arrangement is shown in C, figure 93.

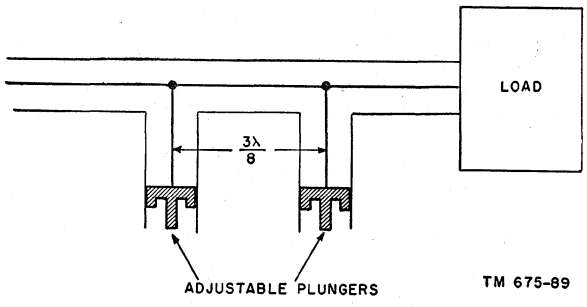


Figure 89. Double stubs on coaxial line.

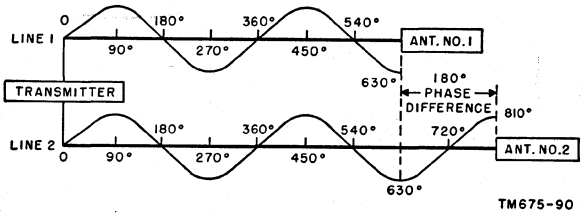


Figure 90. Lines of unequal length for phasing.

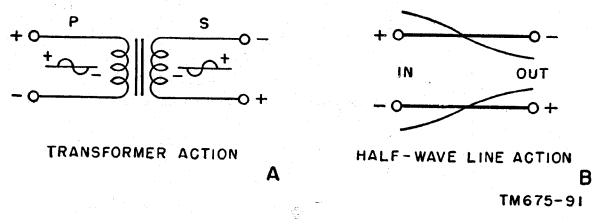


Figure 91. Phase inversion.

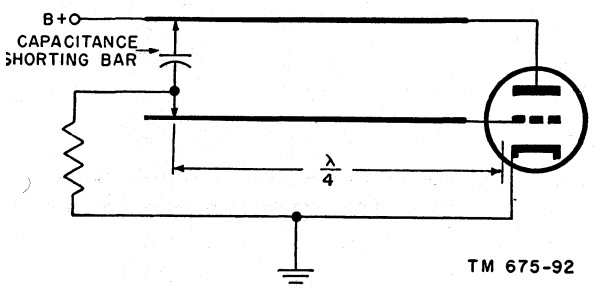


Figure 92. Simplified line oscillator circuit.

Section VIII. ARTIFICIAL TRANSMISSION LINES

81. General

An artificial transmission line (fig. 94) can be thought of as an electrical network, composed of lumped elements of inductance, capacitance, and resistance. The artificial line has characteristics similar to those of an actual transmission line. Although the primary purpose of a transmission line is to guide the transfer of energy from one place to another, the transmission line has characteristics which may be useful for other applications. For example, if a voltage is applied to the input terminals of a line, a definite amount of time passes before the voltage appears at the output terminals. In other words, the line has the ability to delay voltages and currents, and the longer the line the greater the delay. If a voltage must be applied to the grid of a tube 1 or 2 microseconds after it has been applied to another part of the circuit, a length of transmission line can be placed ahead of the grid to give the necessary time lag.

82. Circuits

To avoid the bulk of an actual line, compact artificial lines of (essentially) the same characteristics may be built of coils and capacitors. A circuit for an artificial line is shown in figure 94. The distributed inductance and resistance of the line are lumped in several choke coils, the distributed capacitance is replaced by capacitors, and the conductance is omitted. If the action of a real transmission line is to be closely approximated, the sections must be small

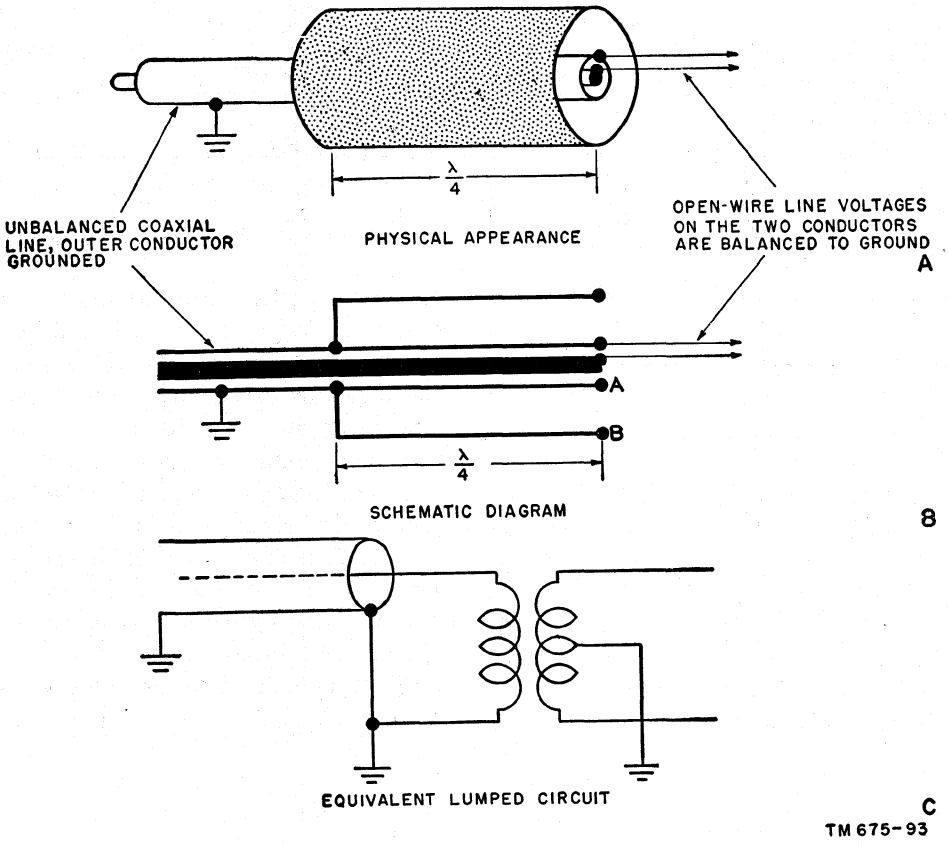


Figure 93. Line-balance converter (bazooka).

and numerous. Usually, from three to eight sections produce the required transmission line action.

83. Other Uses

a. Artificial transmission lines also may be used as filters to block or pass certain frequencies, and as models for laboratory demonstration of transmission line action.

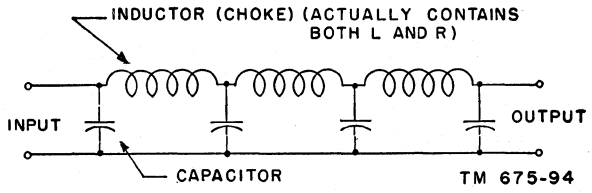


Figure 94. Simple artificial line circuit.

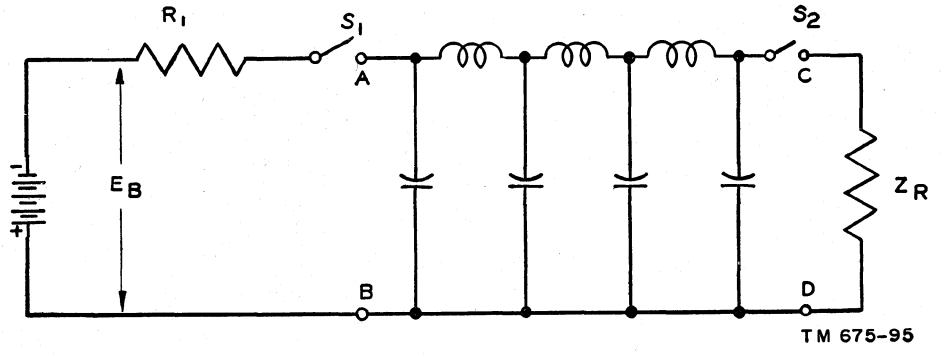


Figure 95. Artificial line as an energy-storing device.

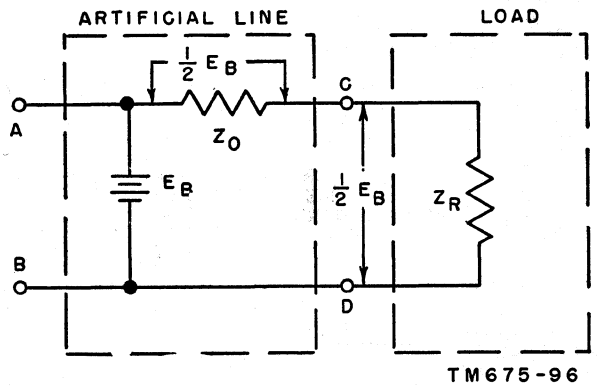


Figure 96. Line discharges through Z_o in series with Z_r.

b. One important use is the storage of energy and the subsequent delivery of the same energy at a *predetermined rate* to form pulses. In figure 95, if switch S_1 is closed, with S_2 open, the artificial line will charge through R to the voltage E_b. The charge is retained by the capacitors. If switch S_1 then is opened and switch S_2 is closed, the line discharges through Z_r.

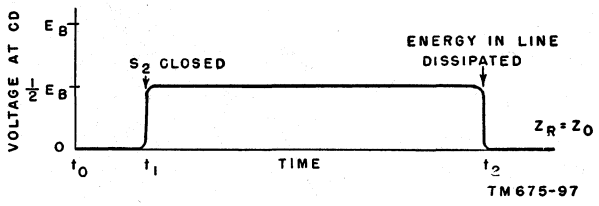


Figure 97. Waveshape of voltage across Z_r.

Since the combination of inductors and capacitors is acting as a transmission line, the voltage appearing at terminals CD is not equal to E_b, as might be expected; the voltage does not decrease as in an ordinary R-C (resistance-capacitance) circuit. The capacitors discharge through the characteristic impedance, Z_o, of the line and through the load impedance, Z_r (fig. 96). If Z_o is made equal to Z_r, the voltage divides equally, so that the voltage across CD is $E_b/2$ as shown in figure 97. When Z_o is equal to Z_r, the voltage across Z_r remains constant and equal to $E_b/2$ value as long as any energy remains in the line.

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